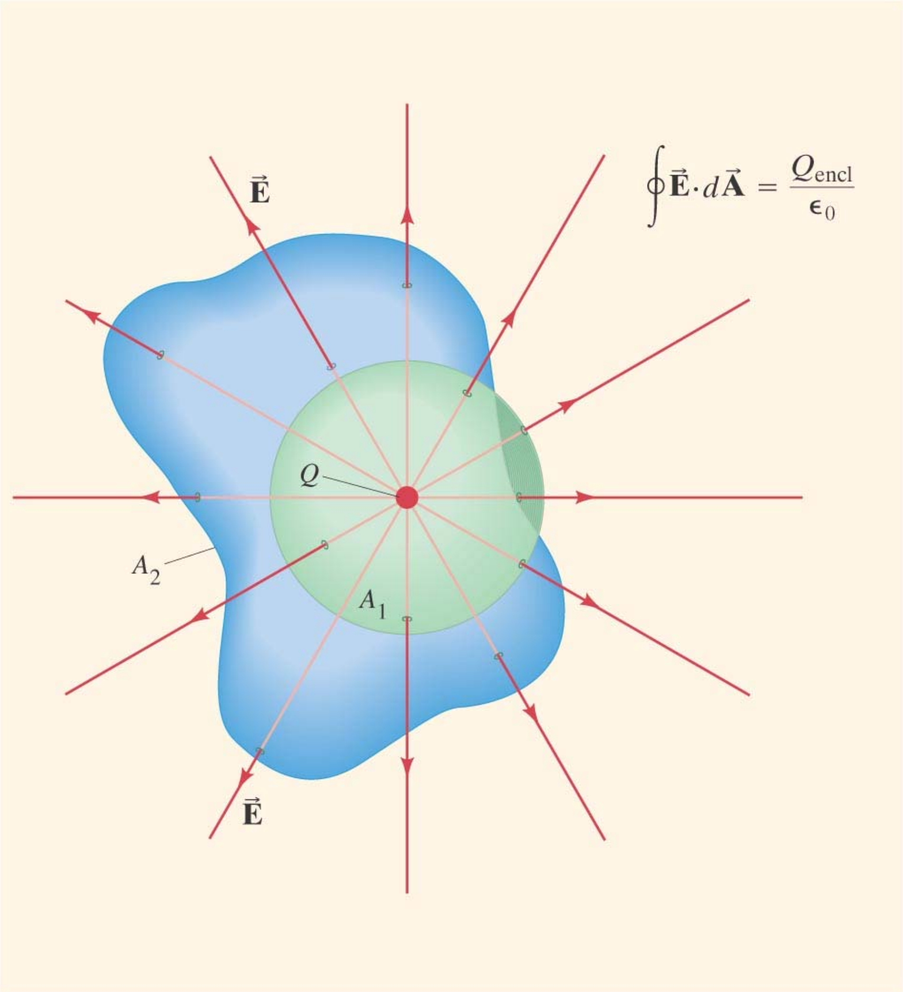
Chapter 22
Gauss's Law



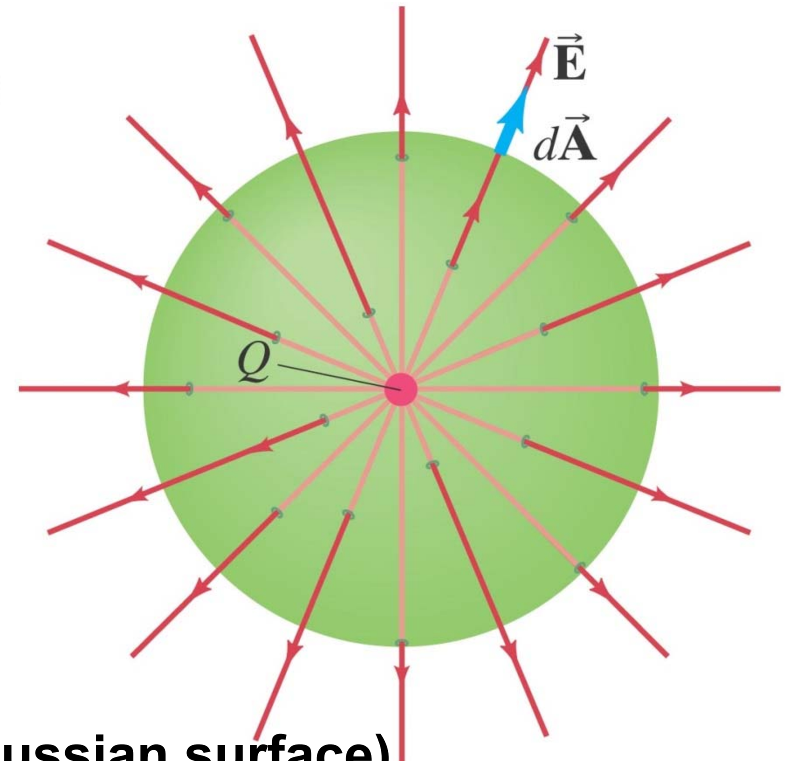
I shot an arrow
Electric Flux analogous to a flock of arrows

The Robbins' Analogy to grasp FLUX Φ concept!

FLUX Φ is a Metaphor for Number of arrows, N, or number of lines of E Penetrating an area! $\Phi \sim N$. Think of E as #arrows/area How many arrows penetrate area! The number is E*A or sum over an area that encloses the charges "shooting" the arrows!

For a point charge, I derive GAUSS'S LAW VIA COULOMB'S LAW

$$\Phi = N = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\, dA = E \oint dA = E(4\pi r^2).$$



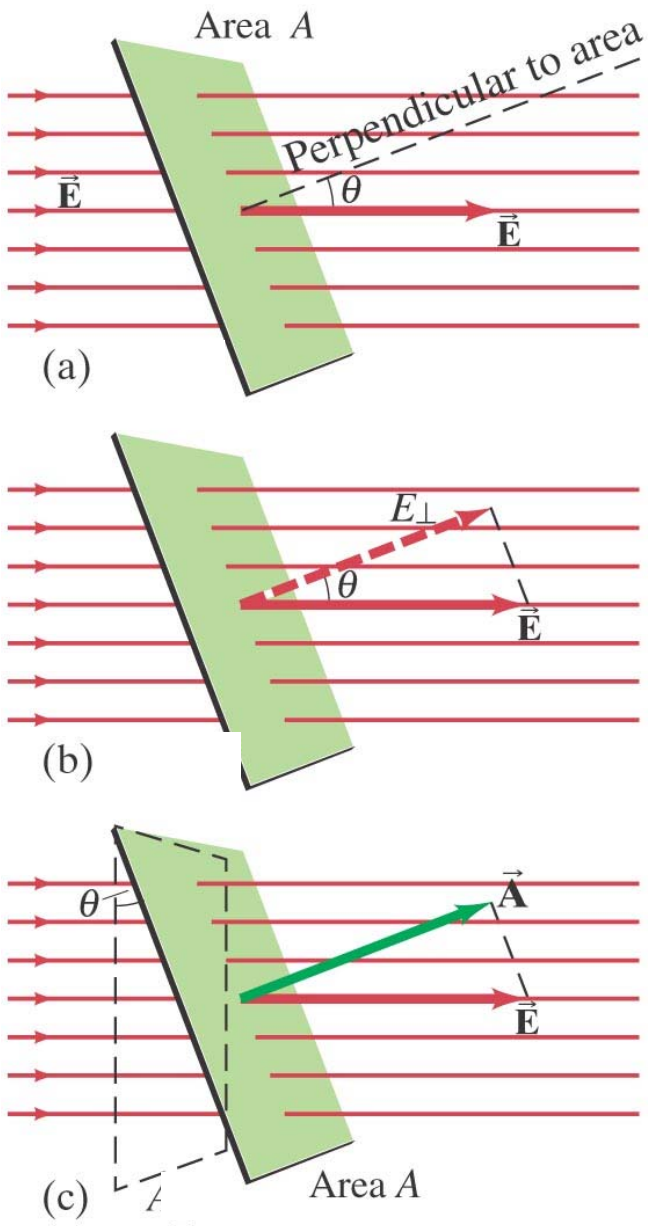
For a Point charge coulombs law gives
$E=kQ/r^2$
So $\Phi = E4\pi r^2 = kQ/r^2 * 4\pi r^2 = 4\pi k Q = Q/\varepsilon_0$

THUS Gauss's law

$$\Phi = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\epsilon_0}.$$

Where Q_{encl} is inside the surface (called a gaussian surface)

22-1 Electric Flux



Electric flux: Φ

$$\Phi_E = E_\perp A = EA_\perp = EA\cos\theta, \qquad [\vec{\mathbf{E}}\text{ uniform}]$$

Remember the sphere

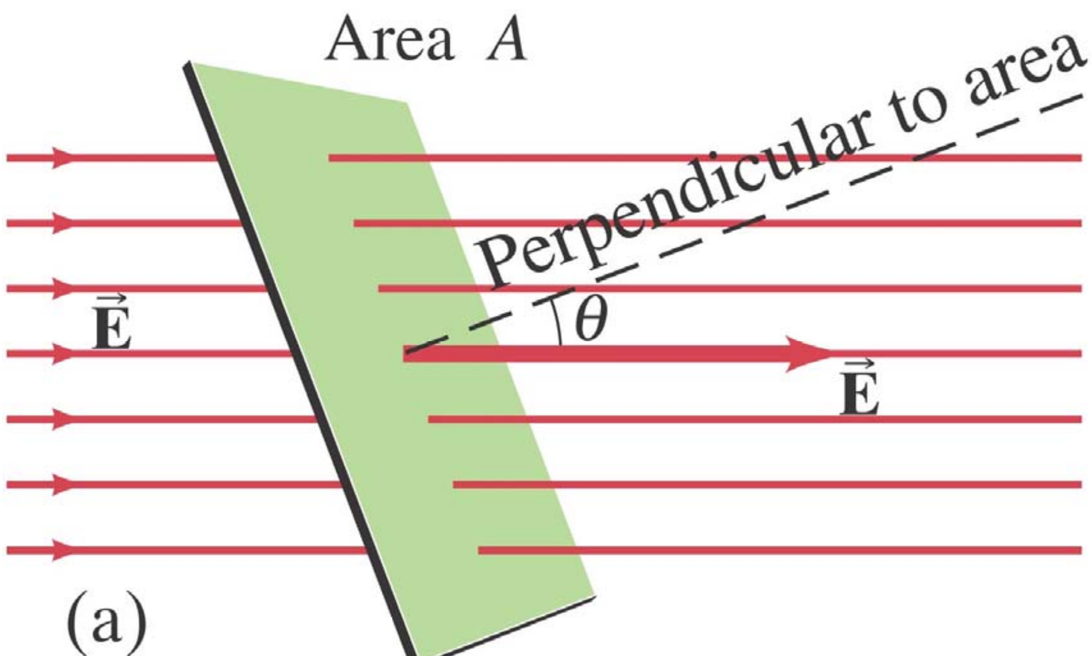
$$\Phi_E = \vec{\mathbf{E}}\cdot\vec{\mathbf{A}}. \qquad [\vec{\mathbf{E}}\text{ uniform}]$$

Electric flux through an area is proportional to the total number of field lines crossing the area.

Tilted surface to E gets less arrows penetrating the Area. If surface is II To E then N=0 going through the Area.

Example 22-1: Electric flux.

Calculate the electric flux through the rectangle shown. The rectangle is 10 cm by 20 cm, the electric field is uniform at 200 N/C, and the angle θ is 30°. NOW in CLASS!



What are The units Of FLUX!

$EA = N/C * m^2$
$= N\text{-}m^2 /C$

$\Phi = 200 N/C * 0.10 m * 0.20 m * \cos(30) = 3.5 N\text{-}m^2 /C$

22-1 Electric Flux

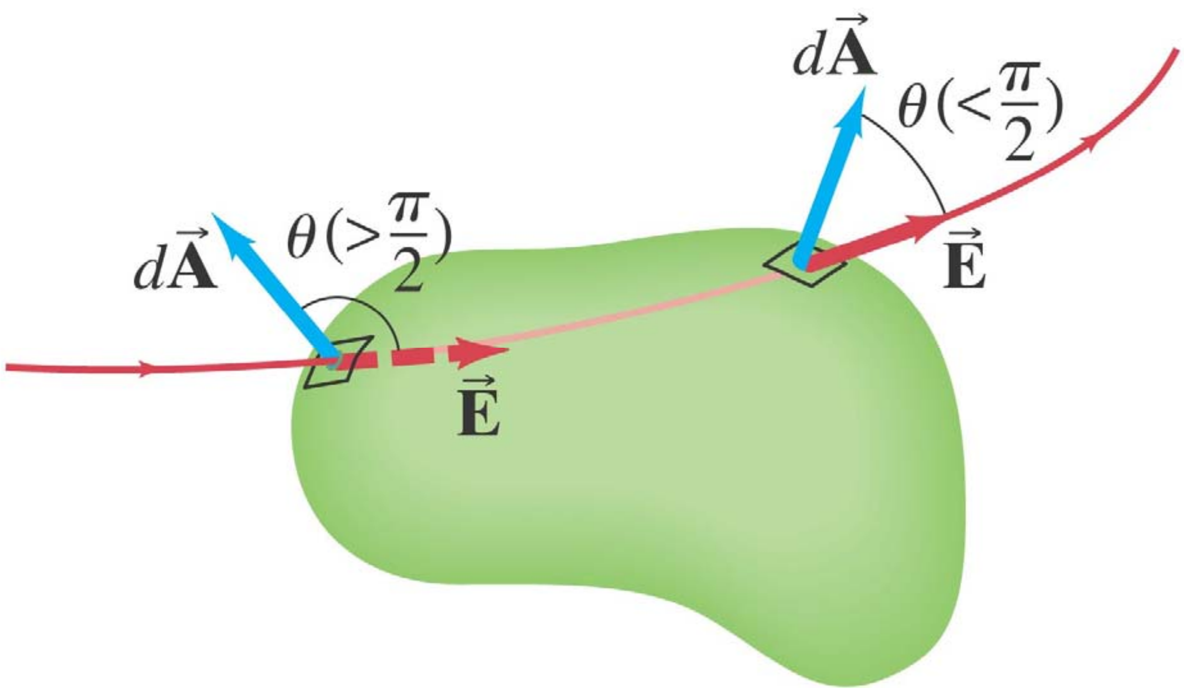
Flux through a closed surface: No Q inside Q_{encl} =0

$$\Phi_E \approx \sum_{i=1}^{n} \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i .$$

$N_{in} - N_{out} = 0!$

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$$

=0!



22-2 Gauss's Law

The net number of field lines through the surface is proportional to the charge enclosed, and also to the flux, giving Gauss's law:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\epsilon_0}.$$

This can be used to find the electric field in situations with a high degree of symmetry. Non-symmetry problems are very difficult!!!

22-2 Gauss's Law applied

For a point charge,

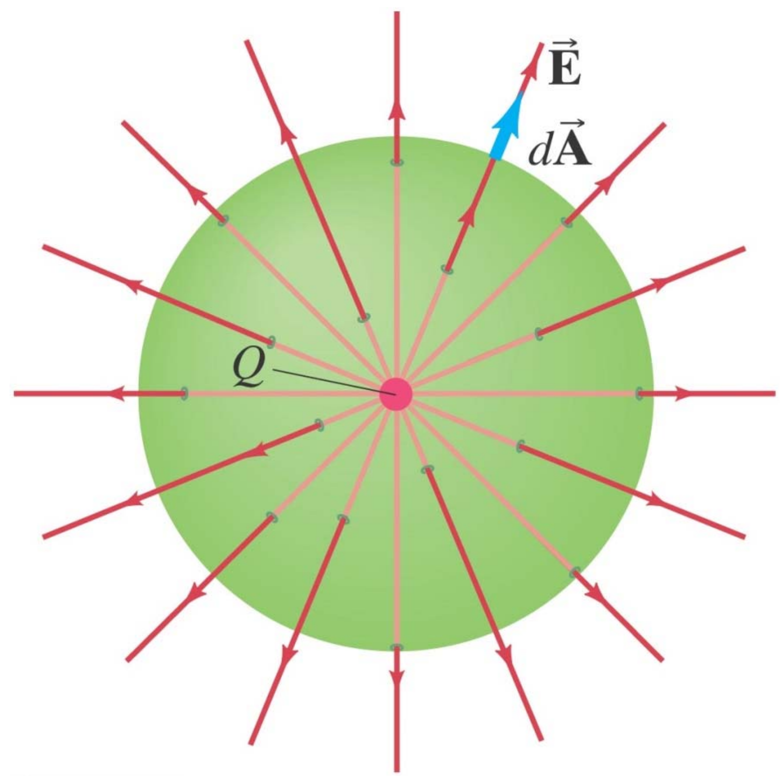
$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\, dA = E \oint dA = E(4\pi r^2).$$

Therefore,

$$\frac{Q}{\epsilon_0} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(4\pi r^2).$$

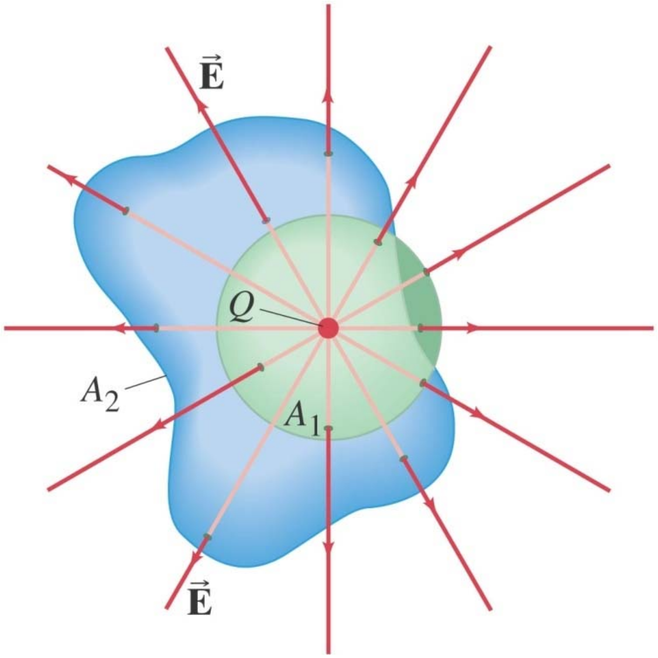
Solving for *E* gives the result we expect from Coulomb's law: AS WE SAW!

$$E = \frac{Q}{4\pi\epsilon_0 r^2}.$$



22-2 Gauss's Law

Using Coulomb's law to evaluate the integral of the field of a point charge over the surface of a sphere surrounding the charge gives:



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,dA = \frac{Q}{4\pi\epsilon_0 r^2}\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}.$$

Looking at the arbitrarily shaped surface A_2, we see that the same flux passes through it as passes through A_1. Therefore, this result should be valid for any closed surface about Q!.

22-2 Gauss's Law

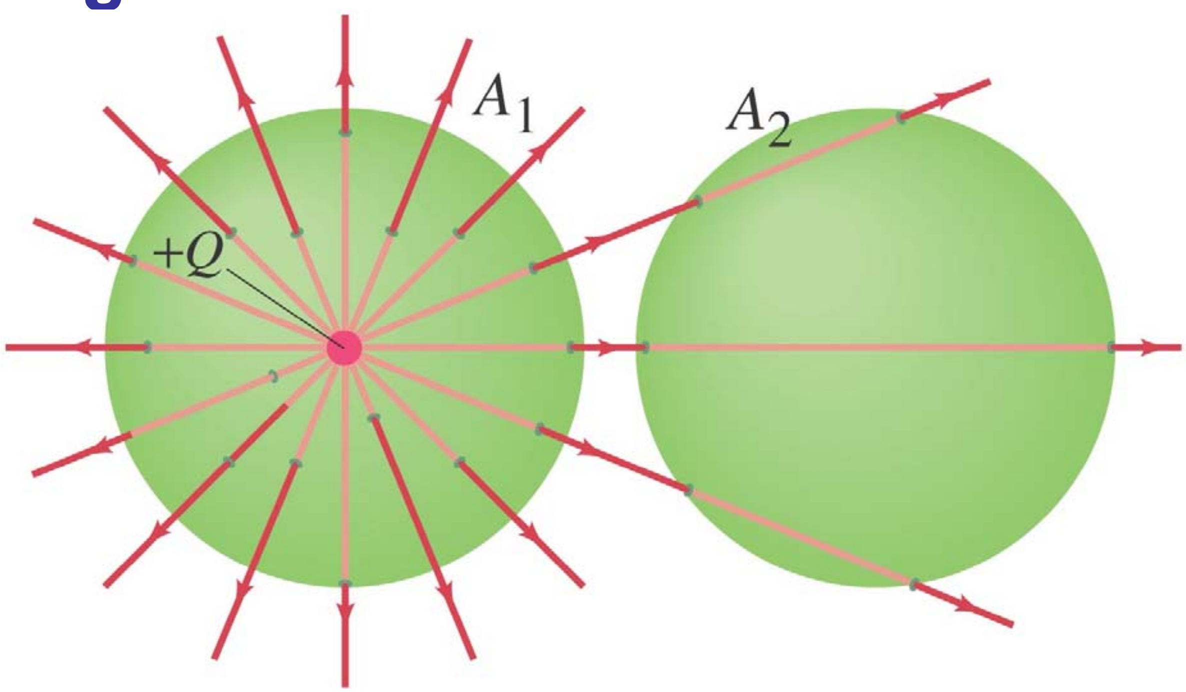
Finally, if a gaussian surface encloses several point charges, the superposition principle shows that:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint \left(\Sigma \vec{\mathbf{E}}_i \right) \cdot d\vec{\mathbf{A}} = \Sigma \frac{Q_i}{\epsilon_0} = \frac{Q_{\text{encl}}}{\epsilon_0}.$$

Therefore, Gauss's law is valid for any charge distribution. Note, however, that it only refers to the field due to charges within the gaussian surface – charges outside the surface will also create fields.

22-2 Gauss's Law

Conceptual Example 22-2: Flux from Gauss's law.Consider the two gaussian surfaces, A_1 and A_2, as shown. The only charge present is the charge Q at the center of surface A_1. What is the net flux through each surface, A_1 and A_2? Gauss's law gives



For A_1 $\Phi = Q/\varepsilon_0$

For A2 $\Phi = 0$

Hand in HW(show all work) understanding flux!

17. If a point charge is located at the center of a cube and the electric flux through one face of the cube is 8.0 Nm^2/C, what is the total flux leaving the cube?

18. A charge q = 4 μC is placed at the origin in a region where there is already a uniform electric field = (200 N/C) . Calculate the flux of the net electric field through a Gaussian sphere of radius R = 20 cm centered at the origin.

19. If the electric flux through a circular area is 10.0 Nm^2/C, what is the electric flux through a circle of double the diameter assuming the orientations of the circles are the same and the electric field is uniform?

20. Charges + Q, −4Q, and + 2Q are placed inside a cubic enclosure, but their positions are not specified. What is the total electric flux passing through the walls of the container? As a formula and if Q=4nC. Value and units!

21. An uniform electric field of magnitude E = 100 N/C is oriented along the positive y–axis. What is the magnitude of the flux of this field through a square of surface area A = 2 m^2 oriented parallel to the yz–plane?

REMINDER: A few words about dQ and charge density distribution which could be constant everywhere (ie uniform) or a function of the geometry (eg. radius of sphere) when we need total charge in Gauss's law

Line density =λ= dQ/dl C/m
if uniform λ =Q/l for total charge Q= λl
we use dQ= λdl in a distribution and Q=integral of last

Area density σ = dQ/dA C/m^2
if uniform σ = Q/A for total charge Q = σA
we use dQ= σdA in a distribution and Q=integral of last

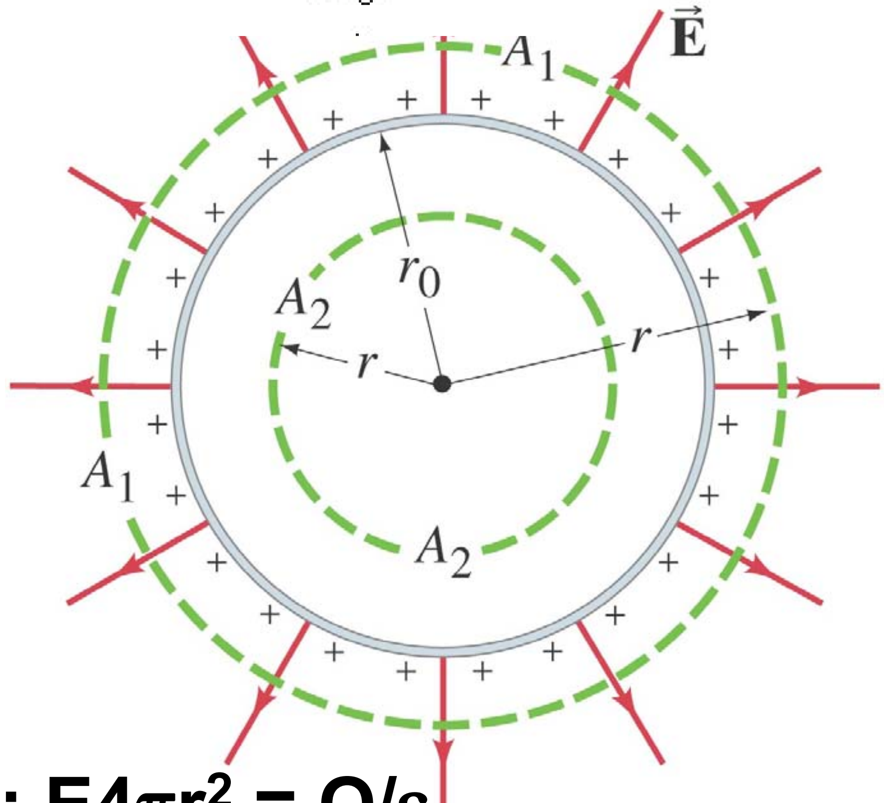
Volume density ρ = dQ/dV C/m^3
if uniform ρ = Q/V for total charge Q= ρdV
we use dQ= ρdV in a distribution and Q=integral of last

22-3 Applications of Gauss's Law

Example 22-3:
Spherical conductor.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\epsilon_0}.$$

A thin spherical shell of radius r_0 possesses a total net charge Q that is uniformly distributed on it. Determine the electric field at points (a) outside the shell, and (b) within the shell. (c) What if the conductor were a solid sphere?



A_1 : $E4\pi r^2 = Q/\varepsilon_0$

$E=(1/4\pi\varepsilon_0)\ Q/r^2$

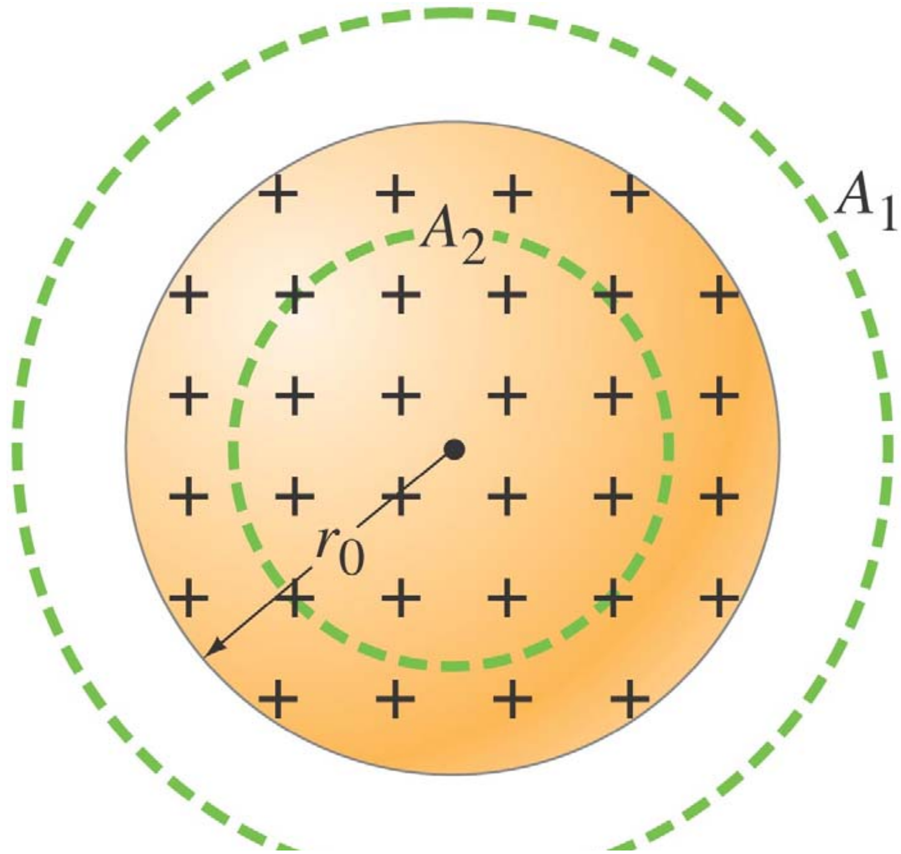
A_2: $E4\pi r^2 = 0$ -> $E = 0$!

PART C??

22-3 Applications of Gauss's Law

Example 22-4: Solid sphere of charge.

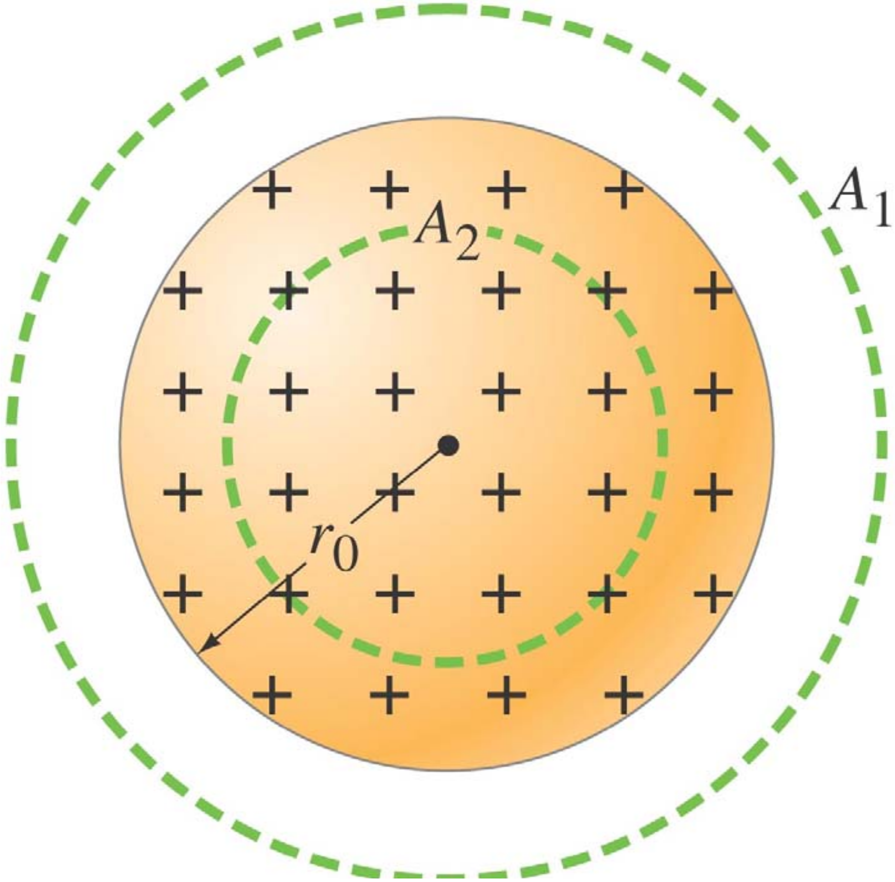
An electric charge Q is distributed uniformly throughout a nonconducting sphere of radius r_0. Determine the electric field (a) outside the sphere ($r > r_0$) and (b) inside the sphere ($r < r_0$).



$$A_1 : \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\epsilon_0}.$$

$$E 4\pi r^2 = Q/\varepsilon_0$$

$$E = (1/4\pi\varepsilon_0)\ Q/r^2$$



$r < r_0$

Charge spreads in Volume so we use

ρ = constant =Q/V

$= Q/(4/3)\pi r_0^3 = Q_{encl}/(4/3)\pi r^3$

$Q_{encl} = r^3/r_0^3 \quad Q$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\epsilon_0} = E 4\pi r^2 \quad \text{on } A_2$$

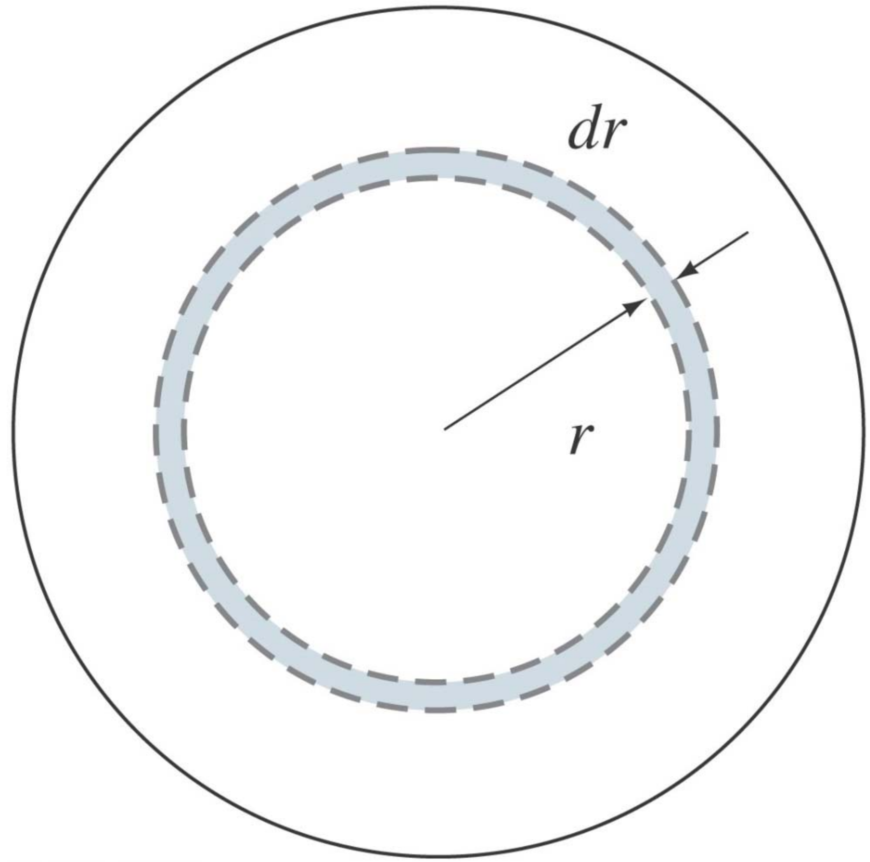
Or for $r<r_0$ $E = (1/4\pi\varepsilon_0)\,(Q/r_0^3)\,r$ after cancellation

Or $E = (\rho/3\varepsilon_0)\,r$

22-3 Applications of Gauss's Law

Example 22-5: Nonuniformly charged solid sphere.

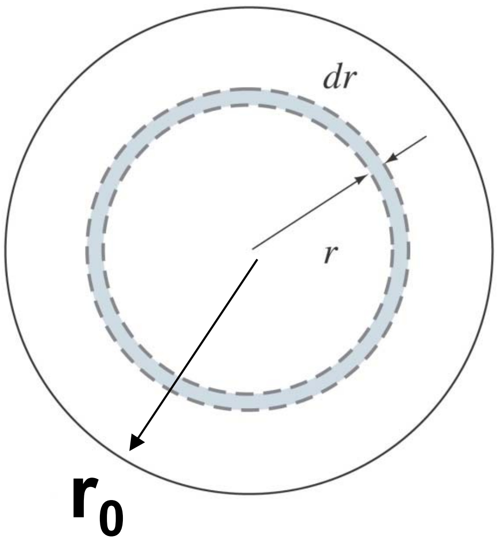
Suppose the charge density of a solid sphere is given by $\rho_E = \alpha r^2$, where α is a constant. (a) Find α in terms of the total charge Q on the sphere and its radius r_0. (b) Find the electric field as a function of r inside the sphere.



ρ_E =dQ/dV to get Q inside

We need to integrate

dV= $4\pi r^2$ dr by symmetry of sphere



VERY IMPORTANT PROBLEM

Find α ? in terms of total Q:

We need to integrate to r_0

where $\rho_E = \alpha r^2$

$\rho_E = dQ/dV$ or $dQ = \rho_E\, dV$

$$Q = \int \rho_E\, dV = \int_0^{r_0} \alpha r^2 4\pi r^2\, dr = 4\pi\alpha \int_0^{r_0} r^4\, dr = (4\pi\alpha/5)\, r_0^5$$

EQ #1 Thus $\alpha = 5Q/4\pi r_0^5$

(b) Find E inside as a f(r)? Q_{encl} is within r!

$$Q_{encl} = \int_o^r \rho_E\, dV = (4\pi\alpha/5)\, r^5 = Q\, r^5/r_0^5 \quad \text{using EQ\#1}$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{encl}}{\epsilon_0} \quad \Rightarrow \quad E 4\pi r^2 = Q r^5 / r_0^5 \varepsilon_0$$

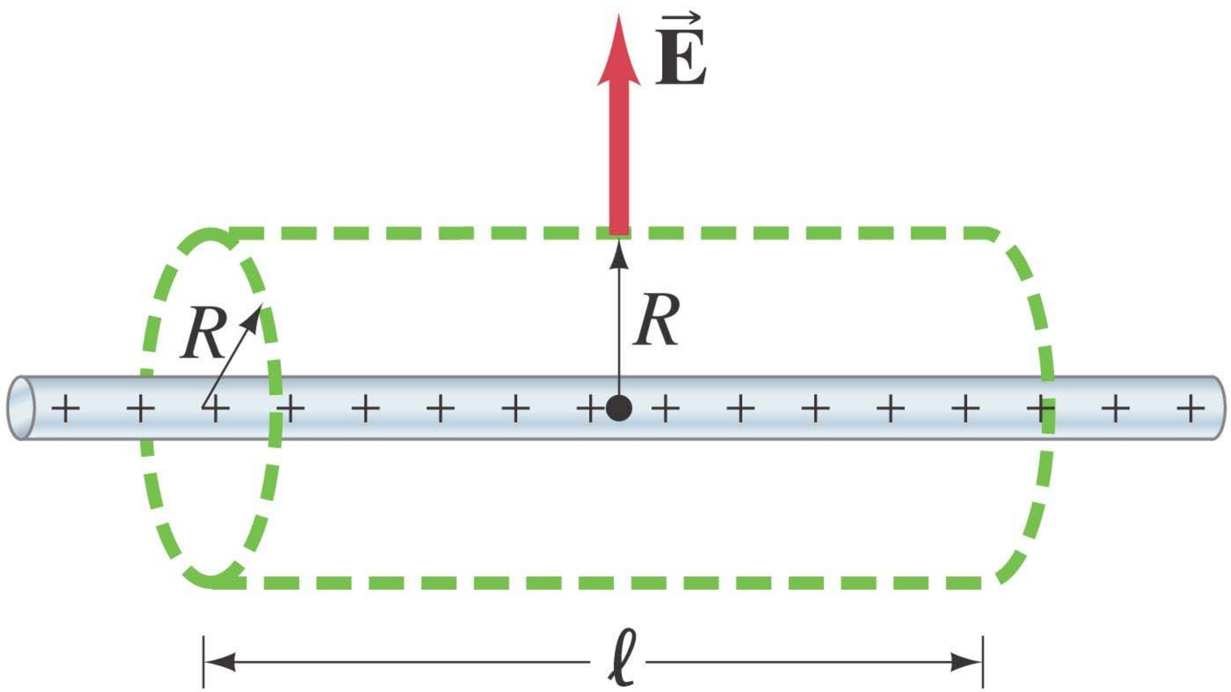
$$E = Q r^3 / 4\pi\varepsilon_0 r_0^5$$

Note $r=0$ $E=0$ and $r=r_0$ $E=Q/4\pi\varepsilon_0 r_0^2$!

22-3 Applications of Gauss's Law

Example 22-6: Long uniform line of charge.

A very long straight wire possesses a uniform positive charge per unit length, λ. =Q/l Calculate the electric field at points near (but outside) the wire, far from the ends.



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\epsilon_0}.$$

$$E 2\pi R l = \lambda\, l/\varepsilon_0$$

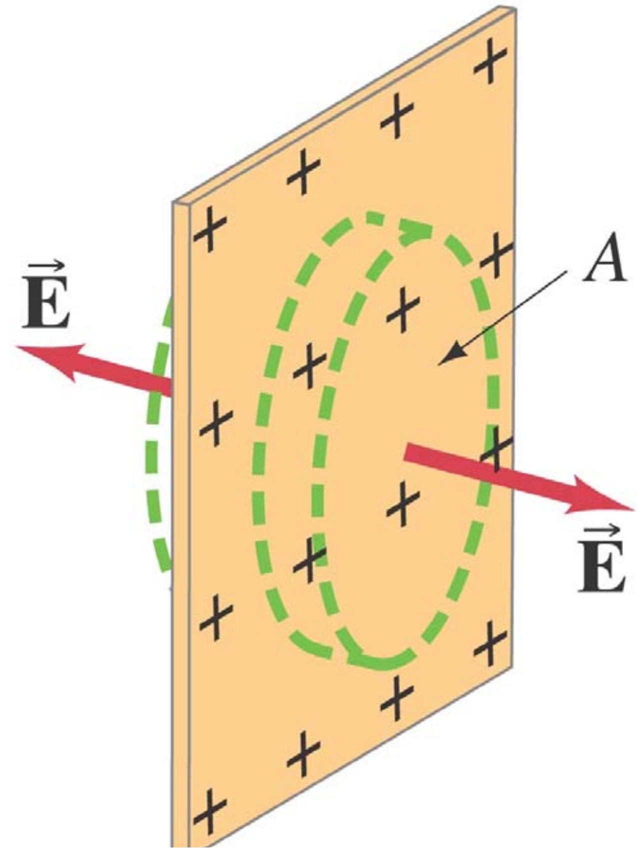
$$E = (1/2\pi\varepsilon_0)\, \lambda\, /R$$

Also done by coloumbs law
In last chapter

22-3 Applications of Gauss's Law

Example 22-7: Infinite plane of charge.

Charge is distributed uniformly, with a surface charge density σ (σ = charge per unit area = dQ/dA) over a very large but very thin nonconducting flat plane surface. Determine the electric field at points near the plane. ($Q_{encl} = \sigma A$)



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}.$$

$2EA = \sigma A / \varepsilon_0$ **$E = \sigma / 2\varepsilon_0$**

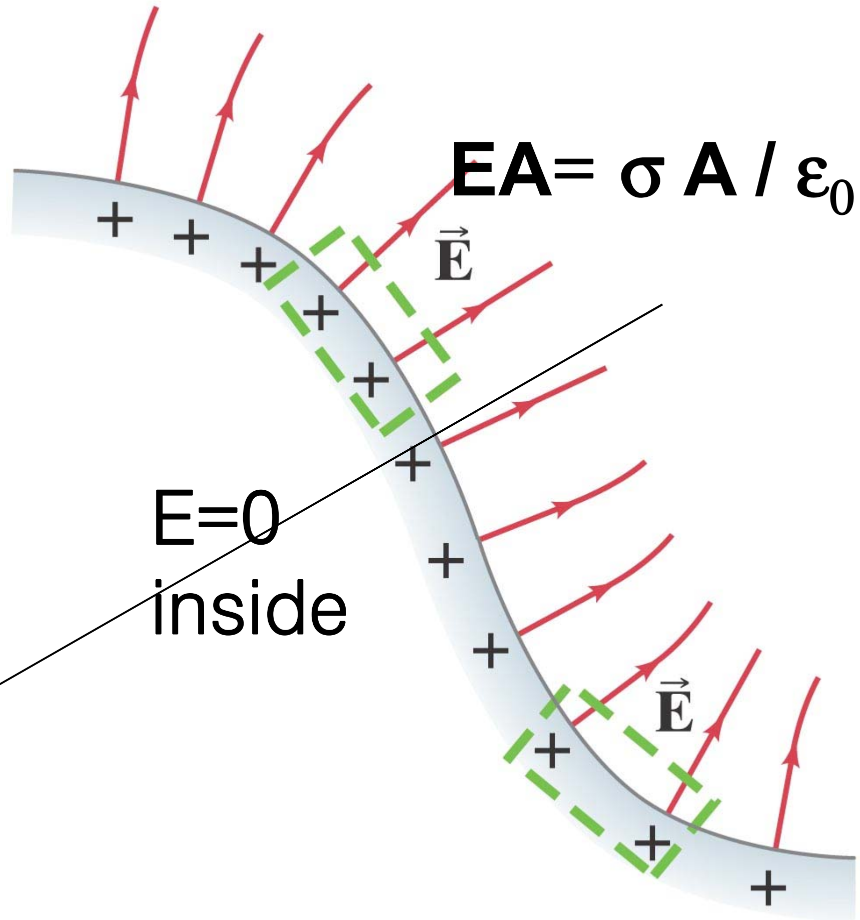
22-3 Applications of Gauss's Law

Example 22-8: Electric field near any conducting surface.

Show that the electric field just outside the surface of any good conductor of arbitrary shape is given by

$$E = \sigma/\varepsilon_0$$

where σ is the surface charge density on the conductor's surface at that point.



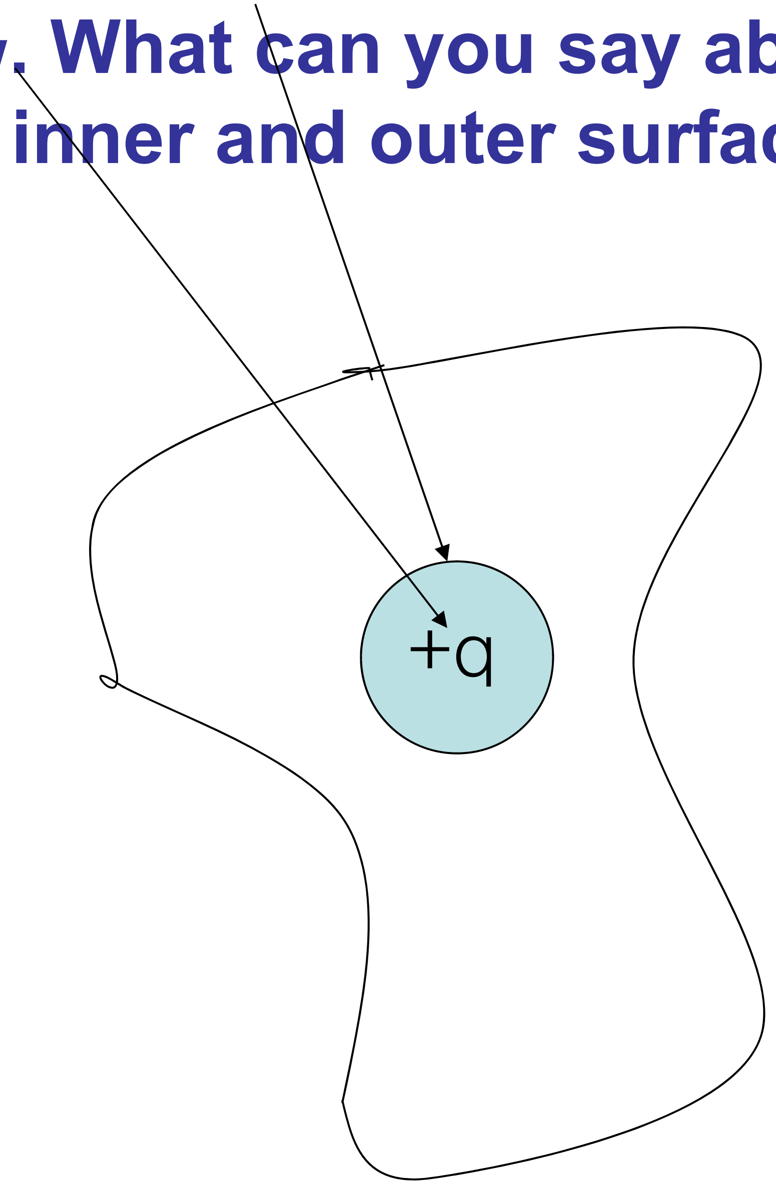
22-3 Applications of Gauss's Law

The difference between the electric field outside a conducting plane of charge and outside a nonconducting plane of charge can be thought of in two ways:

1. The field inside the conductor is zero, so the flux is all through one end of the cylinder. i.e. we use a small cylinder as before

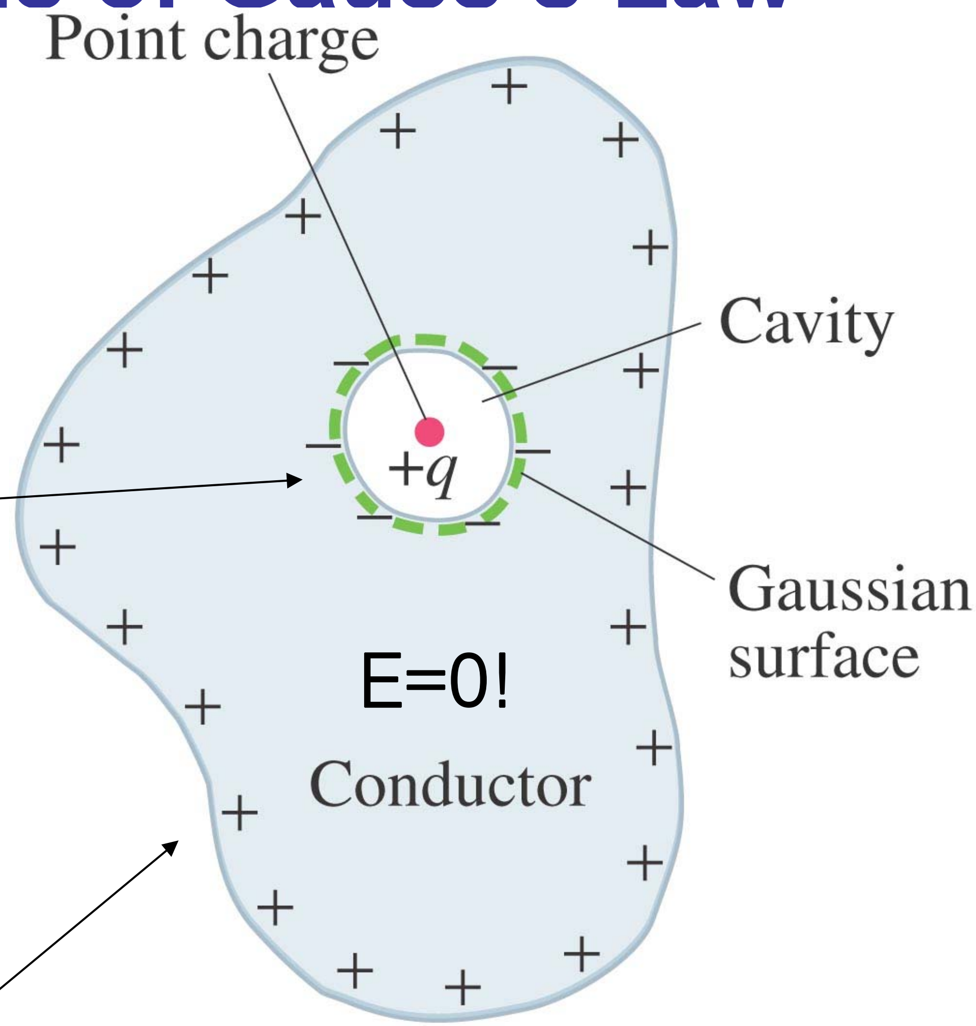
2. The nonconducting plane has a total charge density σ, whereas the conducting plane has a charge density σ on each side, effectively giving it twice the charge density.

Suppose a conductor carries a net charge +Q and contains a cavity, inside of which is a point charge +q. What can you say about the charges on the inner and outer surfaces of the conductor?



22-3 Applications of Gauss's Law

Conceptual Example 22-9: Conductor with charge inside a cavity.



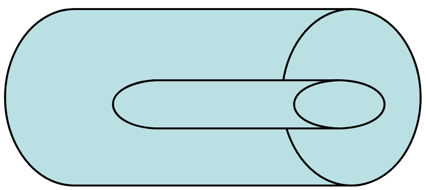
−q must
Be induced
On inner
Cavity wall
Which means
+q is added to
The outside

Or q+Q is now on outer surface
The conductor itself has −q on inside
And Q+q on outside net =Q+q−q=Q!

Hand in HW applications of Gauss's Law show formula answers then quantify!

22. An infinitely long cylinder of radius R = 4 cm carries a uniform charge density = 29 $\mu C/m^3$. Calculate the electric field at distance r = 2 cm from the axis of the cylinder (ie inside) and r=8cm (ie outside).

23. A coaxial cable consists of two long concentric conducting cylindrical shells of radii a and b. The linear charge density on the surface of the inner conductor (eg a wire of radius a!) is + λ and on the outer conductor –λ. Determine E everywhere. Ie 1. a<r<b? 2. r>b? 3. r<a?



24. Charge Q is distributed uniformly over a hollow spherical surface (like the shell of a soccer ball) of radius R. Determine E inside and outside the sphere.(ie r<R and r>R)?

25. Charge is distributed with uniform volume charge density ρ throughout the volume of a sphere of radius R. Determine E everywhere. r<R? r>R?

26. A spherical, non-conducting shell of inner radius r_1= 8 cm and outer radius r_2= 18 cm carries a total charge Q = 15 μC distributed uniformly throughout its volume. A. What is the electric field at a distance r= 14 cm from the center of the shell? HINT:This means we are inside the shell and there is a volume distribution so you have to determine the Q inside the Gaussian surface you draw. Text Example 22- 4 might help

27. A solid sphere of radius R has an electric charge density ρ = br. A. Find b in terms of R and total charge Q on the sphere?. B. Determine the electric field inside and outside the sphere?.
Hint: see text example 22-5

22-3 Applications of Gauss's Law

Procedure for Gauss's law problems:

1. Identify the symmetry, and choose a gaussian surface that takes advantage of it (with surfaces along surfaces of constant field).
2. Draw the surface.
3. Use the symmetry to find the direction of $\vec{E}$.
4. Evaluate the flux by integrating.
5. Calculate the enclosed charge.
6. Solve for the field.