

## Chapter 23 Electric Potential



## 23-1 Electrostatic Potential Energy and Potential Difference



The electrostatic force, here,F=qE is conservative - potential energy can be defined.
$\underset{\substack{\text { Low } \\ \text { poential }}}{ }$ Change in electric potential (U) energy energy is negative of work done Low by the electric force:
$\mathbf{W}=\mathrm{Fd}=q E d$ here and $\Delta \mathbf{U}=-\mathbf{W}$

$$
U_{\mathrm{b}}-U_{\mathrm{a}}=-W=-q E d .
$$

$W$ is the work done on $q($ note + ) by $F$ Remember gravity!!
Note $q>0 \quad U_{a}>U_{b}$ ie $U_{b}=U_{a}-q E d$ $q$ will exchange energy $U$ to $K$ (inetic)

## 23-1 Electrostatic Potential Energy and Potential Difference

Electric potential is defined as potential energy per unit charge:

$$
V_{\mathrm{a}}=\frac{U_{\mathrm{a}}}{q} .
$$

Unit of electric potential: the volt (V):

$$
1 \mathrm{~V}=1 \mathrm{~J} / \mathrm{C} .
$$

## 23-1 Electrostatic Potential Energy and Potential Difference a deeper understanding!

Only changes in potential can be measured, allowing free assignment of $V_{b}=0\left(\mathrm{U}_{\mathrm{b}}=0\right)$ : usually a ground! In figure below q positive charge goes from a to b!

$$
V_{\mathrm{ba}}=\Delta V=V_{\mathrm{b}}-V_{\mathrm{a}}=\frac{U_{\mathrm{b}}-U_{\mathrm{a}}}{q}=-\frac{W_{\mathrm{ba}}}{q}
$$


$\Delta \mathrm{U}=\mathrm{q} \mathrm{V}_{\mathrm{ba}}$ an important connection!
\& $\mathbf{W}_{\mathrm{ba}}=$ work done by $\mathrm{F}=\mathrm{qE}$ on $\mathbf{q} ; \mathrm{W}_{\mathrm{ba}}=q E d$
Potential difference $=\mathrm{V}_{\mathrm{b}}-\mathrm{V}_{\mathrm{a}}=\mathrm{V}_{\mathrm{ba}}=$ voltage!
potential
energy
$\mathrm{V}_{\mathrm{b}}=\mathrm{V}_{\mathrm{a}}-\mathrm{W}_{\mathrm{ba}} / \mathrm{q}=\mathrm{V}_{\mathrm{a}}-\mathrm{qE} / \mathrm{q}$ and $\Delta \mathrm{U}=\mathrm{q} \mathrm{V}_{\mathrm{ba}}$
$V_{b} \operatorname{RECALL} \Delta U+\Delta K=0 * *\left(U_{b}-U_{a}\right)+\left(K_{b}-K_{a}\right)=0$
$q>0$ then $V_{a}>V_{b}$ it gains Potential energy $b$->a losses $a->b$ Again analogous to lifting an object against gravity b->a A gain in potential energy, let it go and you get kinetic from The drop in potential energy. $\operatorname{HERE} \mathrm{U}_{\mathrm{b}}=0 \quad \mathrm{~K}_{\mathrm{a}}=0$ So**-> $-\mathrm{U}_{\mathrm{a}}=-\mathrm{K}_{\mathrm{b}}$ or $\mathrm{K}_{\mathrm{b}}=\mathrm{U}_{\mathrm{a}}$ Potential energy at a shows up as Kinetic Energy at b! or at $b: 1 / 2 \mathrm{mv}^{2}=q \mathrm{~V}_{\mathrm{a}}$ !

## 23-1 Electrostatic Potential Energy and Potential Difference

Conceptual Example 23-1: A negative charge.
Suppose a negative charge, such as an electron, e, is placed near the negative plate at point $b$, as shown here. If the electron is free to move, will its electric potential energy increase or decrease? How will the electric potential change?
charge here


| Low |
| :--- |
| potential |

$V_{\mathrm{b}}$
High PE
for negative
charge here

Careful this can be confusing. Here e drops potential and gains kinetic But Potential difference is the same As in $\mathrm{q}>0$
for negative Since the V's are due to the charge on the plates not to The electron!

## 23-1 Electrostatic Potential Energy and Potential Difference

Analogy between gravitational and electrical potential energy: thinking about this illustration might help clarify the concepts!


## 23-1 Electrostatic Potential Energy and Potential Difference

Electrical sources such as batteries and generators supply a constant potential difference. Here are some typical potential differences, both natural and manufactured:

TABLE 23-1 Some Typical
Potential Differences (Voltages)

| Source | Voltage <br> (approx.) |
| :--- | :--- |
| Thundercloud to ground | $10^{8} \mathrm{~V}$ |
| High-voltage power line | $10^{5}-10^{6} \mathrm{~V}$ |
| Power supply for TV tube | $10^{4} \mathrm{~V}$ |
| Automobile ignition | $10^{4} \mathrm{~V}$ |
| Household outlet | $10^{2} \mathrm{~V}$ |
| Automobile battery | 12 V |
| Flashlight battery | 1.5 V |
| Resting potential across <br> nerve membrane | $10^{-1} \mathrm{~V}$ |
| Potential changes on skin <br> (EKG and EEG) | $10^{-4} \mathrm{~V}$ |

## 23-1 Electrostatic Potential Energy and Potential Difference

Example 23-2: Electron in CRT.
Suppose an electron in a cathode ray tube is accelerated from rest through a potential difference $V_{\mathrm{b}}-V_{\mathrm{a}}=V_{\mathrm{ba}}=$ +5000 V. (a) What is the change in electric potential energy of the electron? (b) What is the speed of the electron ( $m=9.1 \times 10^{-31} \mathrm{~kg}$ ) as a result of this acceleration?


(b) What is the speed of the electron ( $m=9.1 \times 10-31 \mathrm{~kg}$ ) as a result of this acceleration? Easy by Energy conservation $\Delta \mathrm{U}+\Delta \mathrm{K}=0$ or
$\Delta K=1 / 2 m v^{2}-0=-\Delta U=-q V_{b a}$
$v=\left(-2 q V_{b a} / m\right)^{1 / 2}$ note $q<0$ so $v>0$ and real!
Units $(C J / C / k g)^{1 / 2}=(\mathrm{J} / \mathrm{kg})^{1 / 2}=(\mathrm{Nm} / \mathrm{kg})$
$=\left(\mathrm{kgm} / \mathrm{s}^{2} \mathrm{~m} / \mathrm{kg}\right)^{1 / 2}=\left(\mathrm{m}^{2} / \mathrm{s}^{2}\right)^{1 / 2}=\mathrm{m} / \mathrm{s}$ !!!! a velocity

## HAND IN HW. Recall by first Sketch,set up equations, solve algebraically then plug in numbers. All answers in Scientific notation

28. A proton initially at rest falls through a potential difference of $\mathbf{2 5 , 0 0 0} \mathrm{V}$. What speed does it gain?
29.. How much work does the electric field do in moving a proton from a point with a potential of $\mathbf{+ 2 0 0}$ to a point where it is -40 V ?
29. An electron acquires $3.05 \times 10^{-16} \mathrm{~J}$ of kinetic energy when it is accelerated by an electric field from plate A to plate B.
a. What is the potential difference between the plates?
b. Which plate has the higher potential? Explain for credit!
30. The work done by an external force to move a $-8.98 \mu \mathrm{C}$ charge a distance, from a to $b$, a distance of 0.8 m , was $7.50 \times 10^{-4} \mathrm{~J}$.
a. What was the external force?
b. What was the Electric field strength.
c. The charge started from rest and when it reached point $b$ it had a K.E. of $5.67 \times 10^{-4} \mathrm{~J}$. What was the potential difference it went through?

## 23-2 Relation between Electric Potential and Electric Field

The general relationship between a conservative force and potential energy:
Going from a to $\mathbf{b}$ the Pot energy $\Delta \mathbf{U}=-\mathbf{W}=$ word done by F Think Gravity again! $W=-\int \vec{F} \bullet d \vec{l}$

$$
U_{\mathrm{b}}-U_{\mathrm{a}}=-\int_{\mathrm{a}}^{\mathrm{b}} \overrightarrow{\mathbf{F}} \cdot d \vec{l}
$$

Here $F=q E$ or since $V=U / q$


Potential difference or voltage is

$$
V_{\mathrm{ba}}=V_{\mathrm{b}}-V_{\mathrm{a}}=-\int_{\mathrm{a}}^{\mathrm{b}} \overrightarrow{\mathbf{E}} \cdot d \vec{\ell} .
$$

## 23-2 Relation between Electric Potential and Electric Field

The simplest case is a uniform field:
$V_{\mathrm{ba}}=V_{\mathrm{b}}-V_{\mathrm{a}}=-\int_{\mathrm{a}}^{\mathrm{b}} \overrightarrow{\mathbf{E}} \cdot d \vec{\ell}$.
$E$ is a constant here
And thus $\mathrm{V}_{\mathrm{ba}}=\mathrm{E} \int_{a}^{b} d l$
$V_{\mathrm{ba}}=-E d$
Note on units $\mathrm{F}=\mathrm{qE}$ or $\mathrm{E}->\mathrm{N} / \mathrm{C}$ Also $\mathrm{E}=\mathrm{V} / \mathrm{d}$ or $\mathrm{E}->\mathrm{V} / \mathrm{m}$


## 23-2 Relation between Electric Potential and Electric Field

Example 23-3: Electric field obtained from voltage.

Two parallel plates are charged to produce a potential difference of 50 V . If the separation between the plates is 0.050 m , calculate the magnitude of the electric field in the space between the plates. Calculate now!!!

$$
V_{b a}=E d \text { so } E=50 \mathrm{~V} / 0.05 \mathrm{~m}=1000 \mathrm{~V} / \mathrm{m}
$$



23-2 Relation between Electric Potential and Electric Field Example 23-4: Charged conducting sphere

Determine the potential at a distance $r$ from th center of a uniformly charged conducting sphere of radius $r_{0}$ for (a) $r>r_{0}$, (b) $r=r_{0}$, (c) $r$ $r_{0}$. The total charge on the sphere is $Q$.

Note $E$ and dl are parallel and $\mathrm{I}=\mathrm{r}$

$$
\begin{array}{rll}
V_{\mathrm{ba}}=V_{\mathrm{b}}-V_{\mathrm{a}}=-\int_{\mathrm{a}}^{\mathrm{b}} \overrightarrow{\mathbf{E}} \cdot d \vec{\ell} . & \mathrm{E}=\mathrm{kQ} / \mathrm{r}^{2} \\
=-k Q \int_{a}^{b} d r / r^{2}=k Q\left(\frac{1}{r_{b}}-\frac{1}{r_{b}}\right) & \mathbf{V}=\mathbf{k} \mathbf{Q} / \mathbf{r}
\end{array}
$$

$$
r>r_{0} \Rightarrow r_{b} \rightarrow \infty \Rightarrow V_{b} \rightarrow 0 \Rightarrow V_{a}=V=k Q / r=\frac{Q}{4 \pi \varepsilon \text { or }}
$$

$$
r=r_{0} \Rightarrow V=k Q / r_{0}
$$

$$
r<r_{0} \rightarrow \text { inside } \rightarrow E=0 \Rightarrow \therefore V=k Q / r_{0}
$$

inside $r=r_{a}<r_{0} V_{a}=V_{b}=V$ at $r_{0}$ is a constant inside a conductor!

## 23-2 Relation between Electric Potential and Electric Field

The previous example gives the electric potential as a function of d, and compared with the electric field:

$$
V_{\mathrm{ba}}=V_{\mathrm{b}}-V_{\mathrm{a}}=-\int_{\mathrm{a}}^{\mathrm{b}} \overrightarrow{\mathrm{E}} \cdot d \overrightarrow{\boldsymbol{l}} .
$$

NOTE: differential form of above Integral is dV=-EdI
=-Edr in curve implies
$\mathrm{E}=-\mathrm{dV} / \mathrm{dr}$ here: check it out!
Take the negative derivation of $V=k Q / r$ In class now! What do you get!

## 23-2 Relation between Electric Potential and Electric Field

## Example 23-5: Breakdown voltage.

In many kinds of equipment, very high voltages are used. A problem with high voltage is that the air can become ionized due to the high electric fields: free electrons in the air (produced by cosmic rays, for example) can be accelerated by such high fields to speeds sufficient to ionize $\mathrm{O}_{2}$ and $\mathrm{N}_{2}$ molecules by collision, knocking out one or more of their electrons. The air then becomes conducting and the high voltage cannot be maintained as charge flows. The breakdown of air occurs for electric fields of about $3.0 \times 10^{6} \mathrm{~V} / \mathrm{m}$. (a) Show that the breakdown voltage for a spherical conductor in air is proportional to the radius of the sphere, and (b) estimate the breakdown voltage in air for a sphere of diameter 1.0 cm

## IN CLASS!.

## (a)At surface of conductor $V=k Q / r_{0} \& E=k Q / r_{0}{ }^{2}$ Or V $=r_{0} E$

(b) $\mathrm{r}_{0}=0.5 \mathrm{~cm}=5 \times 10^{-3} \mathrm{~m}$ so $V=5 \times 10^{-3} \mathrm{~m} \times 3.0 \times 10^{6} \mathrm{~V} / \mathrm{m}$ $=15000 \mathrm{~V}$

## HAND IN HW. Recall by first set up sketch,equations, solve algebraically then

 plug in numbers. All answers in Scientific notation32. The voltage between two parallel plates separated by a distance of 6.0 cm is 120 V . The electric field between the plates is ?
33. If the electric field is $20 \mathrm{~V} / \mathrm{m}$ in the positive x -direction (reminder it's a vector!) $\mathrm{Pic!} \longrightarrow+x$
a. what is the potential difference between the origin, $(0,0)$, and the point at ( $3.0 \mathrm{~m}, 0.0 \mathrm{~m}$ )?
b. what is the potential difference between the origin, $(0,0)$, and the point at ( $3.0 \mathrm{~m}, 5.0 \mathrm{~m}$ )?
c. what is the potential difference between the origin, $(0,0)$, and the point at ( $-4.0 \mathrm{~m}, 5.0 \mathrm{~m}$ )?
Reminder: potential difference is a scalar!
34. The electric field between two parallel plates connected to a 60 V battery is $1500 \mathrm{~V} / \mathrm{m}$. How far apart are the plates?
35. Estimate the electric field in the membrane wall of a living cell. A wall is typically 12 nm thick and has a potential of 0.11 V across it?

## 23-3 Electric Potential Due to Point Charges an

 approach to distributions of chargeTo find the electric potential due to a point charge, we integrate the field along a field line:
$V_{\mathrm{b}}-V_{\mathrm{a}}=-\int_{r_{\mathrm{a}}}^{r_{\mathrm{b}}} \stackrel{\rightharpoonup}{\mathbf{E}} \cdot d \vec{\ell}=-\frac{Q}{4 \pi \epsilon_{0}} \int_{r_{\mathrm{a}}}^{r_{\mathrm{b}}} \frac{1}{r^{2}} d r=\frac{1}{4 \pi \epsilon_{0}}\left(\frac{Q}{r_{\mathrm{b}}}-\frac{Q}{r_{\mathrm{a}}}\right) . \quad=\mathrm{k}\left(\mathbf{Q} / \mathrm{r}_{\mathrm{b}}-\mathbf{Q} / \mathrm{r}_{\mathrm{a}}\right)$


Looks similar to spherical Conductor but here only A single point charge $\mathbf{Q}$

Setting the potential to zero $\mathrm{V}_{\mathrm{b}}->0 \mathrm{r}_{\mathrm{b}}->\infty \& \mathrm{~V}_{\mathrm{a}}=\mathrm{V}, \mathrm{r}_{\mathrm{a}}=\mathrm{r}$ as we did with the spherical distribution gives the general form of the potential due to a point charge: NOT A VECTOR IT'S A SCALAR! IE - $\mathrm{V}_{\mathrm{a}}=$ $k Q / r_{a}$-> or for each point charge at a distance $r$ from it

V =kQ/r

## 23-3 Electric Potential Due to Point Charges

$$
V=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r} .
$$

$\left[\begin{array}{c}\text { single point charge; } \\ V=0 \text { at } r=\infty\end{array}\right]$

V is an absolute Potential or a Potential difference From $r$ to infinity: so again a distance or $r$ from a point charge Q the Potential (energy/charge at that spot) is $\mathrm{V}=\mathrm{kQ} / \mathrm{r}$. Since it's a scalar other point charges for the same Spot just add up algebraically for the total potential.


For $Q>0$ then potential decreases as $r$ grows


For $Q<0$ the potential increases as $r$ grows.

23-3 Electric Potential Due to Point Charges building further to a distribution of charges
Example 23-7: Potential due to two charges. Calculate the electric potential (a) at point A in the figure due to the two charges shown, and (b) at point $B$.

Since V is a scalar we calculate at each "Point" (A or B in fig) the contribution of each charge $\left(Q_{1} \& Q_{2}\right.$ creating the potential, and of course an Electric field. In other words: The potential is Just a simple sum at the point rather than a vector sum. So again, when we have discrete(individual) charges in a distribution (pattern) and ask for the potential at a point. We add the contributions of each Charge at the | $\begin{array}{c}-26 \mathrm{~cm} \\ Q_{2}=+50 \mu \mathrm{C}\end{array}$ | 26 cm |
| :---: | :---: |
| $Q_{1}=-50$ |  | point in question!


 zand (b) at point B.

Since $V$ for a point charge is Just $=k Q / r$ its easy to sum We have the diagram Now a formula is?
(a) $\mathrm{V}=\mathrm{k} \mathrm{Q}_{2} / \mathrm{r}_{\mathrm{A} 2}+\mathrm{kQ} / \mathrm{r}_{\mathrm{A} 1}$ Formula solution calculate now! Numbers only here, but when you do problems put in units! are $\mathrm{V}=9.0 \times 10^{9}\left(\left(-50 \times 10^{-6} / .6\right)+\left(+50 \times 10^{-6} / .52\right)=7.5 \times 10^{5} \mathrm{~V}\right.$ See TEXT example for alternate calculation! Don't forget!
(b) At B by symmetry what is the answer??
$\mathrm{V}=0$ and every where this is true we have the
Equipotential surface note line labeled $\mathrm{y}!\mathrm{V}=\mathbf{0}$ all along This line.
And you should have seen this DIPOLE field in the lab!

## HAND IN HW. Recall by first Sketch, set up equations, solve algebraically then plug in numbers. All answers in Scientific notation

36. How far from a point charge of 5 C is the potential
a. 50 V ? b. 100 V ?
37. At a certain point in space there is a potential of 820 V . What is the potential energy of a $+4 \mu \mathrm{C}$ charge at that point in space?
38. Two point charges of magnitude $Q_{1}=8.0 \mu \mathrm{C}$ and $\mathrm{Q}_{2}=-8.0 \mu \mathrm{C}$ are situated along the $x$-axis at $x_{1}=3.0 \mathrm{~m}$ and $x 2=-3.0 \mathrm{~m}$, respectively.
a. What is the electric potential at the origin of the $x y$-coordinate system?
b. What is the electric potential on the $x$ axis at $x=6 \mathrm{~m}$ ?
c. What is the electric potential at the point $(3,3)$ in the $x y$ plane?
39. A charge $-q$ is placed at one corner of a square of side $s$, and charges $+q$ are placed at each of the other corners.
a. What is the potential formula solution at the center of the square?
b. What is the value of the potential if the magnitude of $q=8.15 \mu \mathrm{C}$ and the value of side $s=34 \mathrm{~cm}$ ? Show units in the calculated answer for full credit.

23-4 Potential Due to Any Charge Distribution
The potential due to an arbitrary charge distribution can be expressed as a sum or integral (if the distribution is continuous):

$$
\begin{aligned}
V_{\mathrm{a}}=\sum_{i=1}^{n} V_{i} & =\frac{1}{4 \pi \epsilon_{0}} \sum_{i=1}^{n} \frac{Q_{i}}{\gamma_{i \mathrm{a}}} \\
V & =\frac{1}{4 \pi \epsilon_{0}} \int \frac{d q}{i},=\mathbf{k} \int \boldsymbol{d} \boldsymbol{q} / \mathbf{r}
\end{aligned}
$$

So again we note we need ways of getting dq Here we will not delve into much about dq And the relationships to distributions, $\lambda, \sigma \& \rho$

A few words about dq and charge density distribution needed for potential calculations, which could be constant everywhere (ie uniform) or a function of the geometry (eg. radius of a circle.). We need total charge in a distribution for calculating the potential away from a ring and disk only here..

So for an Area density we used in the past $\sigma=d q / d A C / m^{2}$ if uniform $\sigma=Q / A$ for total charge then $Q=\sigma A$. We could use dq= $\sigma d A$ in a distribution and get $Q$ by an integral of last.
But here we will use ratios of $d q / Q=d A / A$ to get a value of dq as the next examples will show.

## 23-4 Potential Due to Any Charge Distribution

Example 23-8: Potential due to a ring of charge.
A thin circular ring of radius $R$ has a uniformly distributed charge $Q$. Determine the electric potential at a point $P$ on the axis of the ring a distance $x$ from its center.

$$
V=\frac{1}{4 \pi \epsilon_{0}} \int \frac{d q}{r},=\frac{\boldsymbol{k}}{\boldsymbol{r}} \int \boldsymbol{d} \boldsymbol{q}
$$

What do you
Note for all dq's $\quad r$ the distance from element dq to $P$ On the ring for By symmetry $r$ for all dq's is constant Point P? Or $r=\left(x^{2}+R^{2}\right)^{1 / 2}$

What happens when you go very far away from the ring in any direction not just on the $x$ axis?


$$
V=\frac{k Q}{\left(x^{2}+R^{2}\right)^{1 / 2}}
$$

$x \gg R$ so $R$ is essentially Neglected next to x !
$\mathrm{V}=\mathrm{kQ} / \mathrm{x}->$ same for all directions So $x=r$ and $V=k Q / r$ same as A point charge!

23-4 Potential Due to Any Charge Distribution

Example 23-9: How to get the Potential due to a charged disk?.

A thin flat disk, of radius $R_{0}$, has a uniformly distributed charge $Q$. Determine the potential at a point P on the axis of the disk, a distance $x$ from its center.

Note: we consider the disk as consisting of many rings with charge dq which can be summed together In

$$
V=\frac{1}{4 \pi \epsilon_{0}} \int \frac{d q}{r} . \quad=\mathbf{k} \int \boldsymbol{d} \boldsymbol{q} / \boldsymbol{r}
$$




Here, dq is on the ring and Distance to $P$, $r$, is
$r=\left(x^{2}+R^{2}\right)^{1 / 2}$
$R$ is the inner radius of the ring.
The text gets dq by ratio of element charge, dq to total charge. Q equail to the ratio of ring area to Disk area. The charge ratio is dq/Q what is the Area of the ring?

Ring area at $R$ (see fig) is the circumference of the circle radius, $R$,
Times its thickness $d R$ which is the
The part of the disk where dq resides.
or ring area $=d A=2 \pi R d R$
Since $d q / Q=d A /$ total $A$ of disk(radius $\left.R_{0}\right)=2 \pi R d R / \pi R_{0}{ }^{2}$
\& now
$d q=2 Q R d R / R_{0}{ }^{2}$ we have transformed the integration of $d q$ to to a sum (integration) over the variable $R$

$$
V=\frac{1}{4 \pi \epsilon_{0}} \int \frac{d q}{r} .=\mathbf{k} \int \boldsymbol{d} \boldsymbol{q} / \boldsymbol{r}
$$

- Solution of integration: we have $k=1 / 4 \pi \varepsilon_{0}$ From the fig-> $r=\left(x^{2}+R^{2}\right)^{1 / 2} \quad x$ is constant! \& $d q=2 Q R d R / R_{0}{ }^{2}$

$$
V=k \int_{0}^{v} d q / r=k \int_{0}^{R_{0}} 2 Q R d R / R_{0}^{2}\left(x^{2}+R^{2}\right)^{1 / 2}=\frac{k 2 Q}{R_{0}^{2}} \int_{0}^{R_{0}} R d R /\left(x^{2}+R^{2}\right)^{1 / 2}
$$

How to solve?
Substitution $U=x^{2}+R^{2}$ so $d U=2 R d R$

$$
\begin{array}{r}
=\frac{k 2 Q}{R_{0}^{2}} \int U^{-1 / 2} d U=\left.\frac{k 2 Q}{R_{0}^{2}} U^{1 / 2}\right|_{0} ^{R_{0}}=\left.\frac{k 2 Q}{R_{0}^{2}}\left(x^{2}+R^{2}\right)^{1 / 2}\right|_{0} ^{R_{0}} \\
V=\frac{Q}{2 \pi \varepsilon_{0} R_{0}^{2}}\left[\left(x^{2}+R^{2}\right)^{1 / 2}-x\right] \quad \begin{array}{l}
\text { The latter is the text version } \\
\text { What does this equation become } \\
\text { When } \mathbf{x} \gg \mathbf{R}_{0} ? \text { See HW }
\end{array}
\end{array}
$$

HAND IN HW. Recall by first Sketch, set up equations, solve algebraically then plug in numbers. All answers in Scientific notation 40.Show the math behind the solution of the text for $V$ Example 23-9 when $x \gg R_{0}$ then $\mathrm{V}=\mathrm{Q} / 4 \pi \varepsilon_{0} \mathrm{x}$
41. A hole of radius $R$ is cut from the center of a uniformly charged disk of radius $2 R$. Determine the potential on the axis of the disk a distance $x$ away.

$42 . A 12 \mathrm{~cm}$ thin ring carries a uniformly distributed $+15 \mu \mathrm{C}$ charge. A small 7.5 g sphere with a charge of $+3.0 \mu \mathrm{C}$ is placed at the center of the ring.
a. What will most likely Happen?
b. The small sphere is given a very small push towards the +x axis.


Nhat will its speed be 2 m from the ring?
Recall potential energy is $\mathrm{U}=\mathrm{qV}$ and that the conservation of energy can lead to the speed of the small sphere! Solve algebraically first for velocity v?

## 23-5 Equipotential Surfaces



An equipotential is a line or surface over which the potential is constant.

Electric field lines are perpendicular to equipotentials.

The surface of a conductor is an equipotential.

## 23-5 Equipotential Surfaces

Example 23-10: Point charge equipotential surfaces.

For a single point charge with $Q=4.0 \times 10^{-9} \mathrm{C}$, sketch the equipotential surfaces (or lines in a plane containing the charge) corresponding
 to $V_{1}=10 \mathrm{~V}, V_{2}=20 \mathrm{~V}$, and $V_{3}=30 \mathrm{~V}$.

V=kQ/r so r=kQ/V for various V's! Note larger V-> r smaller Will this fit on a page?

## 23-5 Equipotential Surfaces

Equipotential surfaces are always perpendicular to field lines; they are always closed surfaces (unlike field lines, which begin and end on charges).


## 23-5 Equipotential Surfaces

A gravitational analogy to equipotential surfaces is the topographical map - the lines connect points of equal gravitational potential (altitude).


23-6 Electric Dipole Potential not stressed by me but if you are Chem or BIO majior check it out

The potential due to an electric dipole is just the sum of the potentials due to each charge, and can be calculated exactly. For distances large compared to the charge separation: see the text calculation $\mathrm{r} \gg$ |

$$
V=\frac{1}{4 \pi \epsilon_{0}} \frac{Q \ell \cos \theta}{r^{2}}=\frac{1}{4 \pi \epsilon_{0}} \frac{p \cos \theta}{r^{2}}
$$



## 23-7 E Determined from V (important! )

If we know the field, we can determine the potential by integrating from the original energy work relation.

$$
V_{\mathrm{ba}}=V_{\mathrm{b}}-V_{\mathrm{a}}=-\int_{\mathrm{a}}^{\mathrm{b}} \overrightarrow{\mathbf{E}} \cdot d \vec{l}
$$

Inverting this process, if we know the potential as a function of space, we can find the field by differentiating: Here in the direction I!

$$
\mathrm{dV}=-\mathrm{E} \bullet \mathrm{dl}=-\mathrm{E}_{\mid} \mathrm{dl}
$$

$$
E_{\ell}=-\frac{d V}{d \ell} \cdot \quad \mathrm{dV} / \mathrm{dl}=\text { gradient of } \mathrm{V} \quad \underset{\mathrm{dl}}{\mathrm{E}_{\mathrm{I}}} \mathrm{E}_{\mathrm{l}}=\mathrm{E} \cos \theta
$$



$$
\begin{aligned}
& \text { V can be a } V(x, y, z) \\
& \text { Recall: } E=E_{x} i+E_{y} j+E_{z} k \\
& \text { And dI }=d x i+d y j+d z k \\
& d V=-E \bullet d I=-E_{x} d x+-E_{y} d y+-E_{z} d z
\end{aligned}
$$

General differential form for a $\mathbf{V}(\mathbf{x}, \mathbf{y}, \mathbf{z})$

changing is. $\mathrm{dV}=\frac{\delta V}{\delta x} \mathrm{dx}+\frac{\delta V}{\delta y} \mathrm{dy}+\frac{\delta V}{\delta \mathrm{z}} \mathrm{dz}-\rightarrow$ or

$$
E_{x}=-\frac{\partial V}{\partial x}
$$

$$
E_{y}=-\frac{\partial V}{\partial y}
$$

$$
E_{z}=-\frac{\partial V}{\partial z} .
$$

What is $E$ for a potential field $V(x, y, z)$ given as $V(x, y, z)=2 x^{2}+8 y^{2} z+2 z^{2}$ Ignoring units what is the value of $E$ (vector)? le the electric vector field $E(x, y, z)$ ?

$$
\begin{aligned}
& \text { recall } \mathrm{E}=\mathrm{E}_{\mathrm{x}} \mathrm{i}+\mathrm{E}_{\mathrm{y}} \mathrm{j}+\mathrm{E}_{\mathrm{z}} \mathbf{k}=-\frac{\delta V}{\delta x} \mathbf{i}-\frac{\delta V}{\delta y} \mathrm{j}-\frac{\delta V}{\delta z} \mathbf{k} \quad \text { or } \\
& \mathrm{E}_{\mathrm{x}}=-\frac{\delta V}{\delta x}=-4 \mathrm{x} \quad \mathrm{E}_{\mathrm{y}}=-\frac{\delta V}{\delta y}=-16 \mathrm{yz} \quad \mathrm{E}_{\mathrm{z}}=-\frac{\delta V}{\delta z}=-8 \mathrm{y}^{2}-4 \mathrm{z} \\
& \mathrm{E}(\mathrm{x}, \mathrm{y}, \mathrm{z})=-4 \mathrm{xi}-16 \mathrm{yzj}-\left(8 \mathrm{y}^{2}+4 \mathrm{z}\right) \mathbf{k}
\end{aligned}
$$

SEE section 23-7 on the above problem and how units are Treated for each component though in the end The unit of $E$ is $V / m$ from the above relation or $N / C$ from $F=q E$

HAND IN HW. Recall by first Sketch, set up equations, solve algebraically then plug in numbers. All answers in Scientific notation
43. If the potential field is given by $V=x y^{2}-3 z-2 x z$, then what is the electric field $E(x, y, z)$ ?
44. The electric potential of a charge distribution is given by the equation $V(x, y, z)=3 x^{3} y^{2}+x y z^{2}-2 z^{2} x$, where $x, y, z$ are measured in meters and $V$ is measured in volts.
a. Find the Equation for the Electric field vector (use $\mathbf{i}, \mathbf{j}, \mathbf{k}$ )?
b. Calculate the magnitude of the electric field vector at the position $(x, y, z)=(1.0,-2.0,3.0) ?$
45. Given the solution of $V$ along the $x$ axis of a ring of charge,

As before $\quad \boldsymbol{V}=\frac{\boldsymbol{k} \boldsymbol{Q}}{\left(\boldsymbol{x}^{2}+\boldsymbol{R}^{2}\right)^{1 / 2}} \quad$ Derive $\mathrm{E}_{\mathrm{x}}=$ ?

## 23-8 Electrostatic Potential Energy; the Electron Volt

The potential energy of a charge in an electric potential is $U=q V$. To find the electric potential energy of two charges, imagine bringing each in from infinitely far away. The first one takes no work, as there is no field. To bring in the second one, we must do work due to the field of the first one; this means the potential energy of the pair is:

$$
U=Q_{2} V=\frac{1}{4 \pi \epsilon_{0}} \frac{Q_{1} Q_{2}}{r_{12}} .
$$

23-8 Electrostatic Potential Energy; the Electron Volt

One electron volt (eV) is the energy gained by an electron moving through a potential difference of one volt: U= qV
$Q_{e}=e=1.6 \times 10^{-19} \mathrm{C} \quad \mathrm{W}=1.6 \times 10^{-19} \mathrm{C} \times 1 \mathrm{~V}=1 \mathrm{eV}$ units $C V=C J / C=J$

$$
1 \mathrm{ev}=1.6 \times 10^{-19} \mathrm{~J}
$$

The electron-volt (eV) is often a much more convenient unit than the joule for measuring the energy of individual particles.
III. $m_{p}=1.67 \times 10^{-27} \mathrm{~kg} \quad E_{p}=m_{p} c^{2}=1.67 \times 10^{-27} \mathrm{~kg} \mathrm{x}$
$\left(3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)^{2}=1.50 \times 10^{-10} \mathrm{~J}->/ 1.6 \times 10^{-19} \mathrm{~J} / \mathrm{ev}=$
$9.39 \times 10^{8} \mathrm{ev}$ or $\mathrm{m}_{\mathrm{p}} \sim 939 \mathrm{MeV} / \mathrm{c}^{2}\left(\right.$ act $938.27 \mathrm{MeV} / \mathrm{c}^{2}$ )

## 23-9 Cathode Ray Tube: TV and Computer Monitors, Oscilloscope

A cathode ray tube contains a wire cathode that, when heated, emits electrons. A voltage
source causes the electrons to travel to the anode.


Battery

## 23-9 Cathode Ray Tube: TV and Computer Monitors, Oscilloscope

## The electrons can be steered using electric or magnetic fields.



## 23-9 Cathode Ray Tube: TV and Computer Monitors, Oscilloscope

Televisions and computer monitors (except for LCD and plasma models) have a large cathode ray tube as their display. Variations in the field steer the electrons on their way to the screen.


## 23-9 Cathode Ray Tube: TV and Computer Monitors, Oscilloscope

An oscilloscope displays an electrical signal on a screen, using it to deflect the beam vertically while it sweeps horizontally.


HAND IN HW. Recall by first Sketch, set up equations, solve algebraically then plug in numbers. All answers in Scientific notation
46. An electron is accelerated from rest through a potential difference of 2000 volts giving it a kinetic energy of ?
a. joules ?
b. eV?
47. What is the energy in electronvolts of an electron with a speed of 4 $\times 10^{6} \mathrm{~m} / \mathrm{s}$ ?
48. In a cathode ray tube used in the old color TV sets an electron is accelerated from rest through a potential difference of 25 kV .
A. How much is the energy the electron gains in eV?
B. What is the velocity of the electron after being accelerated?
49. A neutron in an atom has a mass of ? And by the equation of You know who? $\mathrm{E}=\mathrm{mc}^{2}$ so what is the energy contained in the neutron expressed in eV's?

