

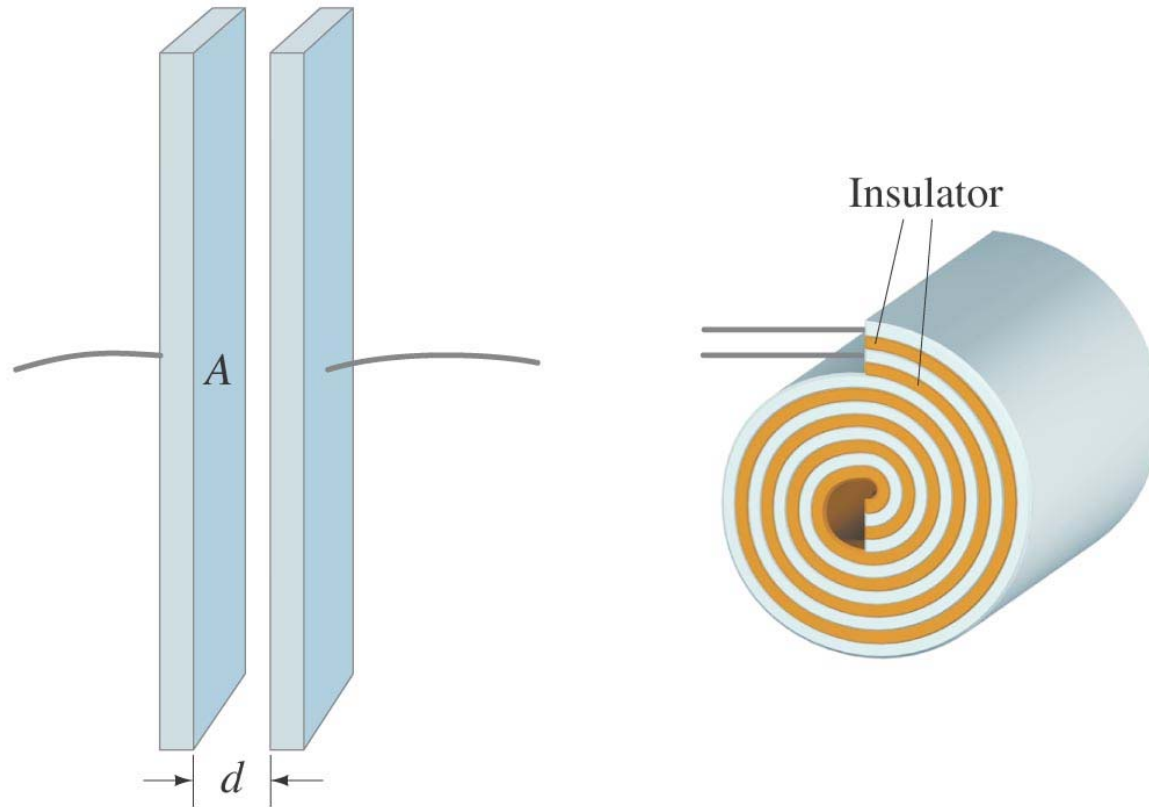
Chapter 24

Capacitance, Dielectrics, Electric Energy Storage



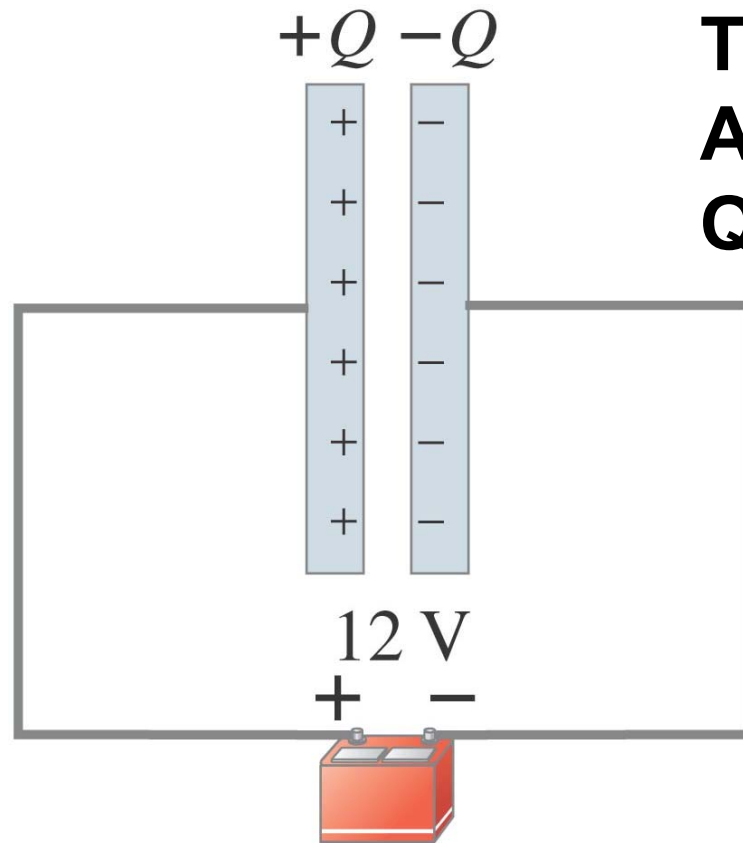
24-1 Capacitors

A capacitor consists of two conductors that are close but not touching. A capacitor has the ability to store electric charge.



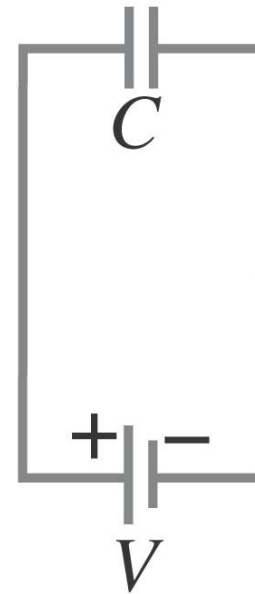
24-1 Capacitors

Parallel-plate capacitor connected to battery. (b) is a circuit diagram.



(a)

**q moves out from
The battery and it takes
A little time for the full
Q to establish itself**



(b)

24-1 Capacitors

When a capacitor is connected to a battery, the charge on its plates is proportional to the voltage:

$$Q = CV.$$

The quantity C is called the capacitance.

Unit of capacitance: the farad (F):

1 F = 1 C/V. was an enormous value

24-2 Determination of Capacitance

For a parallel-plate capacitor as shown, the field between the plates was shown to be $E = \sigma / \epsilon_0$ and $\sigma = Q/A$ we get

$$E = Q / \epsilon_0 A.$$

Integrating along a path between the plates gives the potential difference: use 180 to Change sign

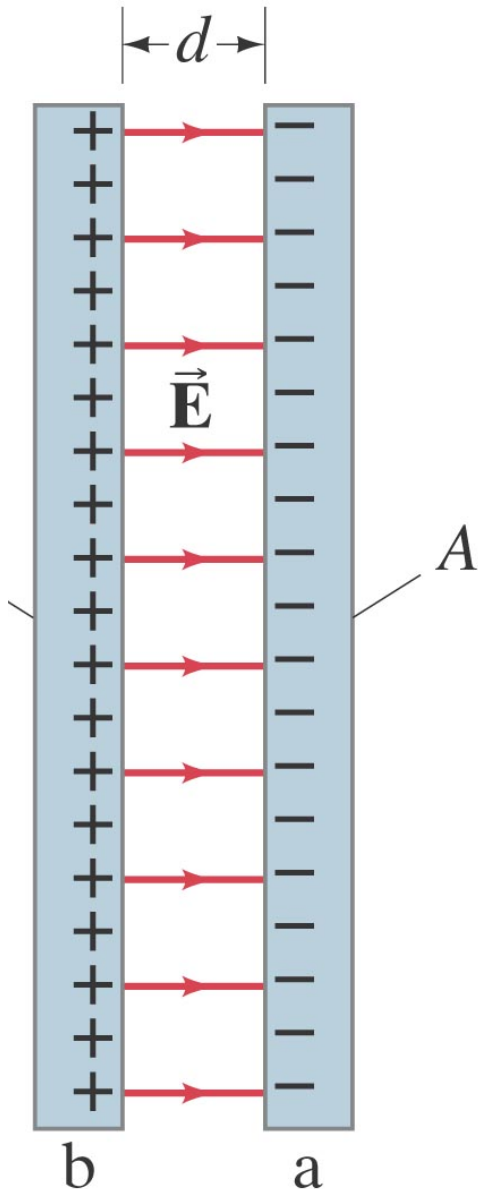
$$V = V_{ba} = V_b - V_a = - \int_a^b E \cdot dl = - \int_a^b E dl \cos 180 = + \int_a^b E dl =$$

$$Ed = Qd / (\epsilon_0 A)$$

With $V_{ba} = Qd / \epsilon_0 A$ then

$$C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}.$$

[parallel-plate capacitor]



24-2 Determination of Capacitance

Example 24-1: Capacitor calculations.

(a) Calculate the capacitance of a parallel-plate capacitor whose plates are 20 cm × 3.0 cm and are separated by a 1.0-mm air gap. (b) What is the charge on each plate if a 12-V battery is connected across the two plates? (c) What is the electric field between the plates? (d) Estimate the area of the plates needed to achieve a capacitance of 1 F, given the same air gap d .

a. $C = \epsilon_0 A/d$ b. $Q = CV$ c. $E = V/d$

d. Solve for A $A = Cd/\epsilon_0$

Value is size of a city for this parallel plate capacitor
so C's where usually pF and μF up to mF in the past

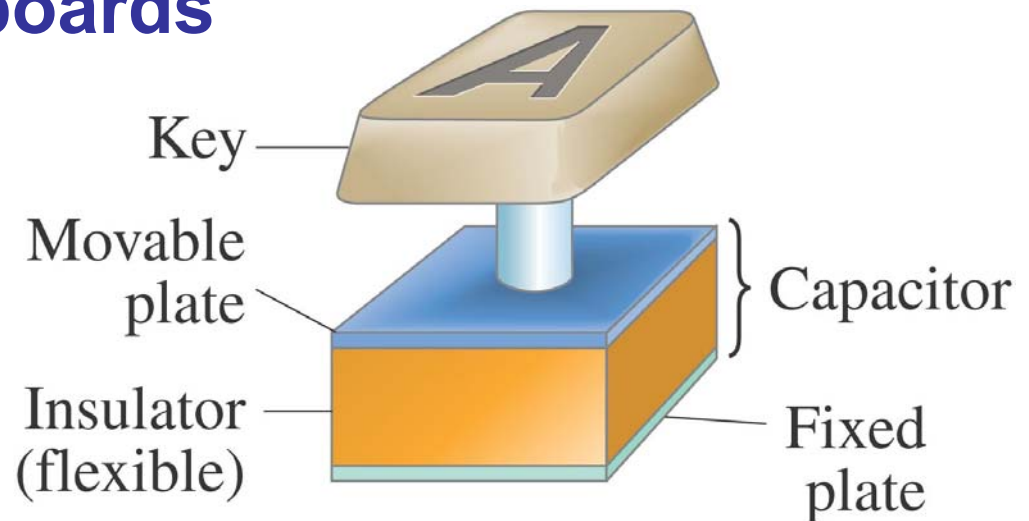
24-2 Determination of Capacitance

Capacitors are now made with capacitances of 1 farad or more, but they are not parallel-plate capacitors. Instead, they are activated carbon, which acts as a capacitor on a very small scale. The capacitance of 0.1 g of activated carbon is about 1 farad.

Activated Carbon high porosity results in Enormous effective Areas -> $1\text{g} = 1000\text{ m}^2$

Some computer keyboards

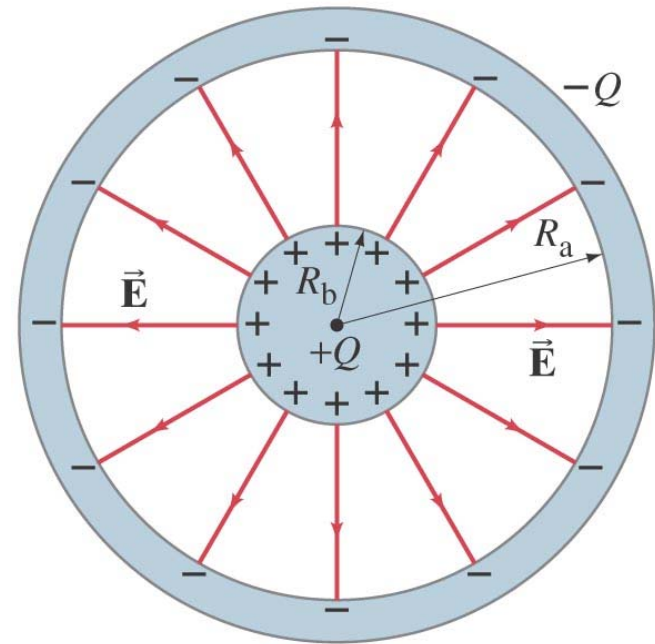
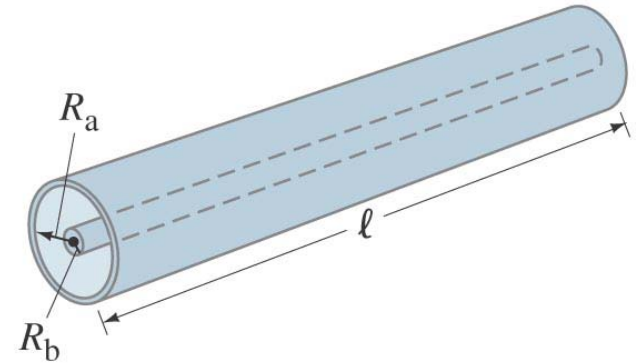
use capacitors; depressing the key changes the capacitance, which is detected in a circuit.



24-2 Determination of Capacitance

Example 24-2: Cylindrical capacitor.

A cylindrical capacitor consists of a cylinder (or wire) of radius R_b surrounded by a coaxial cylindrical shell of inner radius R_a . Both cylinders have length ℓ which we assume is much greater than the separation of the cylinders, so we can neglect end effects. The capacitor is charged (by connecting it to a battery) so that one cylinder has a charge $+Q$ (say, the inner one) and the other one a charge $-Q$. Determine a formula for the capacitance.



C of a cylinder

Since $C=Q/V$ we need V like || plate

$$V = V_{ba} = V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{l}$$

E for long line of charge is done on page 597

$E=2k\lambda/r$ from a gaussian surface radius, r

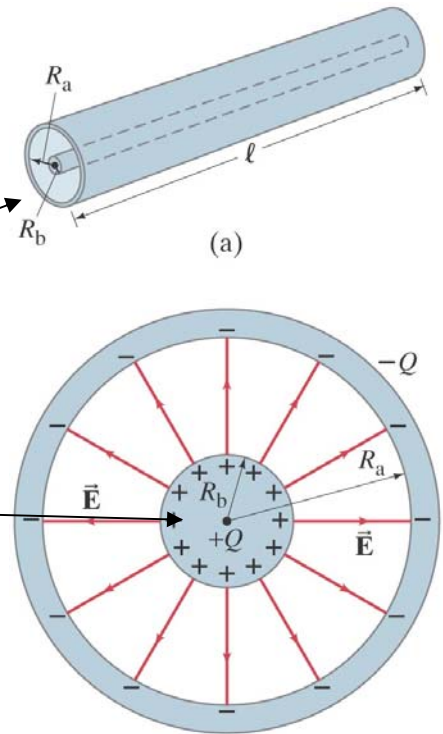
and. $\lambda=Q/l$ so $E=2kQ/lR$ for points

between the cylinders Thus for V we get with

$dl=dR$ ie along the E field lines

$$V = -Q2k/l \int_{R_A}^{R_B} dR/R = -Q2k/l \ln(R_b/R_a) = (Q/2\pi\epsilon_0 l) \ln R_a/R_b$$

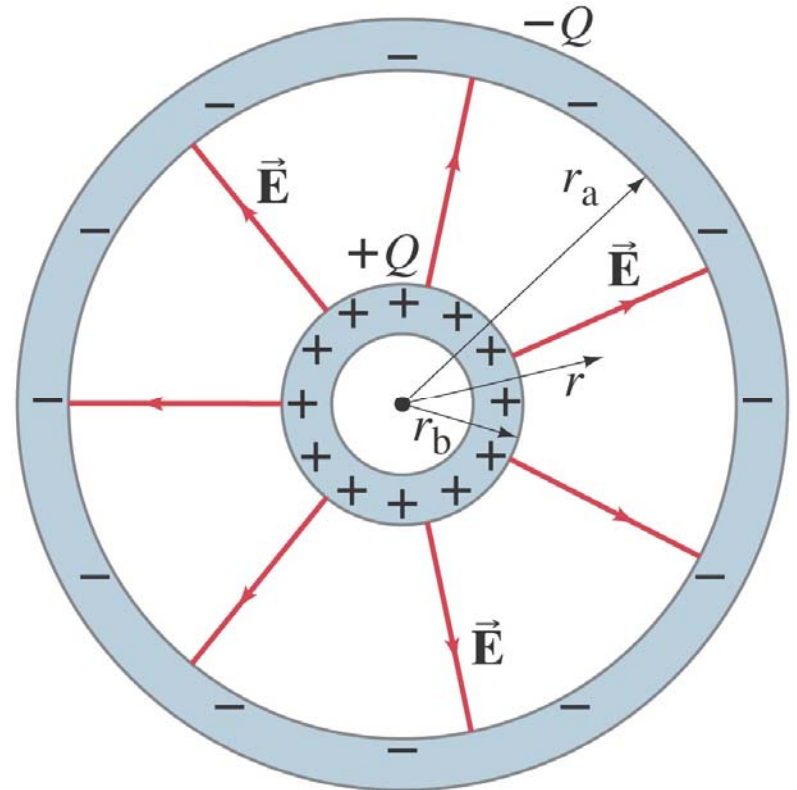
$$C=Q/V=2\pi\epsilon_0 l / \ln(R_a/R_b) \quad C/l = 2\pi\epsilon_0 / \ln(R_a/R_b)$$



24-2 Determination of Capacitance

Example 24-3: Spherical capacitor.

A spherical capacitor consists of two thin concentric spherical conducting shells of radius r_a and r_b as shown. The inner shell carries a uniformly distributed charge Q on its surface, and the outer shell an equal but opposite charge $-Q$. Determine the capacitance of the two shells.



C for a spherical capacitor

From Gauss' law we saw

$E = kQ/r^2$ so you should be able

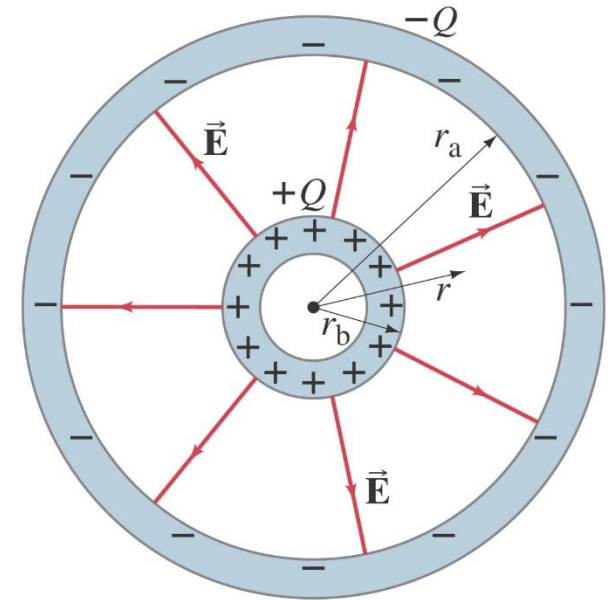
To show that from

$$V = V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

We get

$$V = -Qk \int_{r_a}^{r_b} dr / r^2 = Qk \left(\frac{r_a - r_b}{r_a r_b} \right)$$

$$\text{and } C = Q/V = 4\pi\epsilon_0 (r_a r_b / (r_a - r_b))$$



HAND IN HW. Recall by first Sketch, set up equations, solve algebraically then plug in numbers. All answers in Scientific notation

50. A parallel plate capacitor of plate area 0.04 m^2 and plate separation 0.25 mm is charged to 24 V . Determine the charge on a plate and the electric field between the plates.

51. What is the capacitance per unit length (F/m) of a coaxial cable whose inner conductor has a 1.0 mm diameter and the outer cylindrical sheath has a 5.0 mm diameter. Assume space between is filled with air?

52. An electric field of $4.80 \times 10^5 \text{ V/m}$ is desired between two parallel plates, each of area 21.0 cm^2 and separated by 0.250 cm of air. What charge must be on each plate?

24-3 Capacitors in Series and Parallel

Capacitors in parallel have the same voltage across each one. The equivalent capacitor is one that stores the same charge when connected to the same battery:

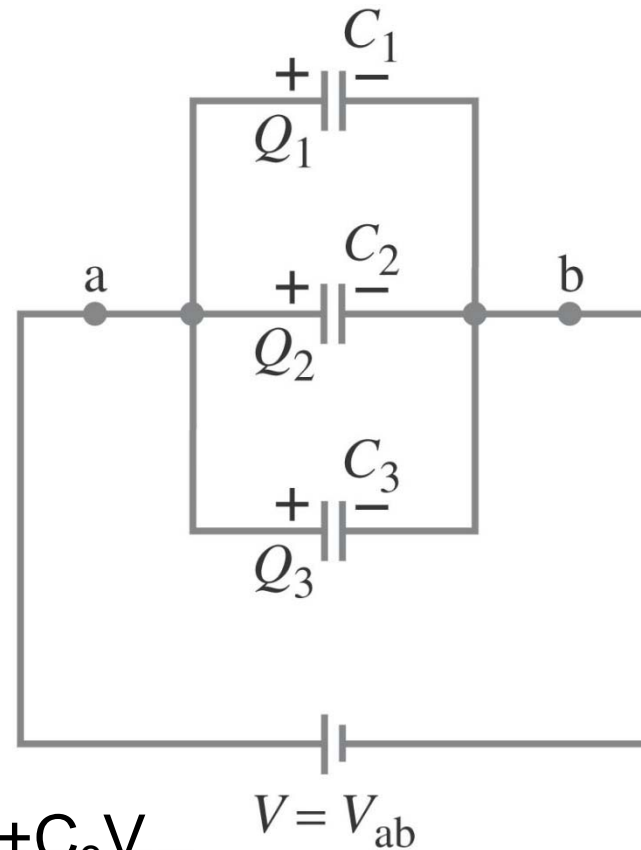
Here V is the same across all C 's
Total $Q = Q_1 + Q_2 + Q_3$

Since $Q = CV$ in general

We have $Q_1 = C_1 V$ etc

$$\begin{aligned} \text{So } Q &= Q_1 + Q_2 + Q_3 = C_1 V + C_2 V + C_3 V \\ &= (C_1 + C_2 + C_3) V = Q = C_{\text{eq}} V \end{aligned}$$

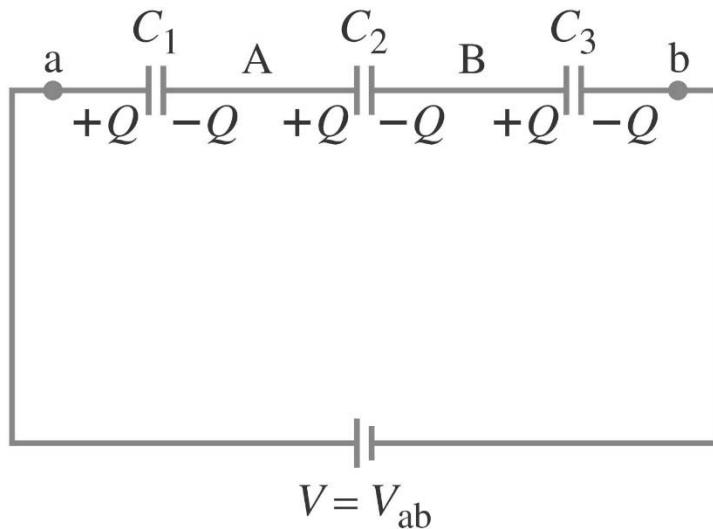
$$C_{\text{eq}} = C_1 + C_2 + C_3.$$



[parallel]

24-3 Capacitors in Series and Parallel

Capacitors in series have the same charge. In this case, the equivalent capacitor has the same charge across the total voltage drop. Note that the final formula is for the inverse of the capacitance and not the capacitance itself!



Here we see that $V = V_1 + V_2 + V_3$
since in general $V = Q/C$ and all Q 's
Are the same we get for a C_{eq}

$Q/C_{eq} = Q/C_1 + Q/C_2 + Q/C_3$ factor out the Q we get

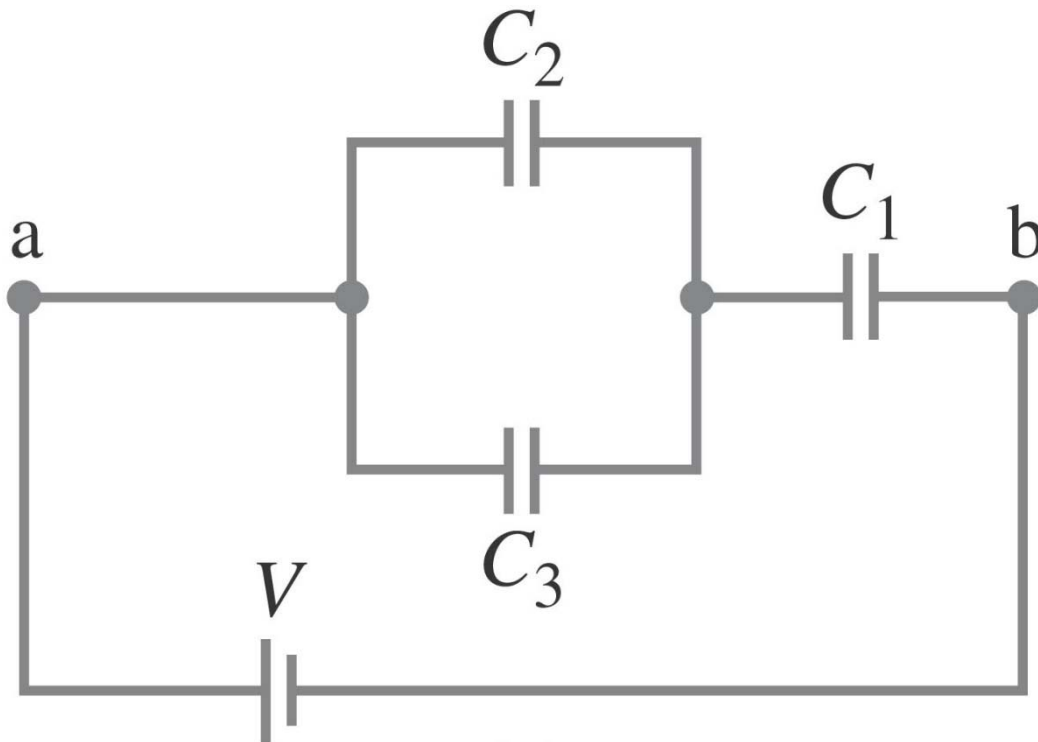
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \quad [\text{series}]$$

24-3 Capacitors in Series and Parallel

Example 24-5: Equivalent capacitance.

Determine the capacitance of a single capacitor that will have the same effect as the combination shown.

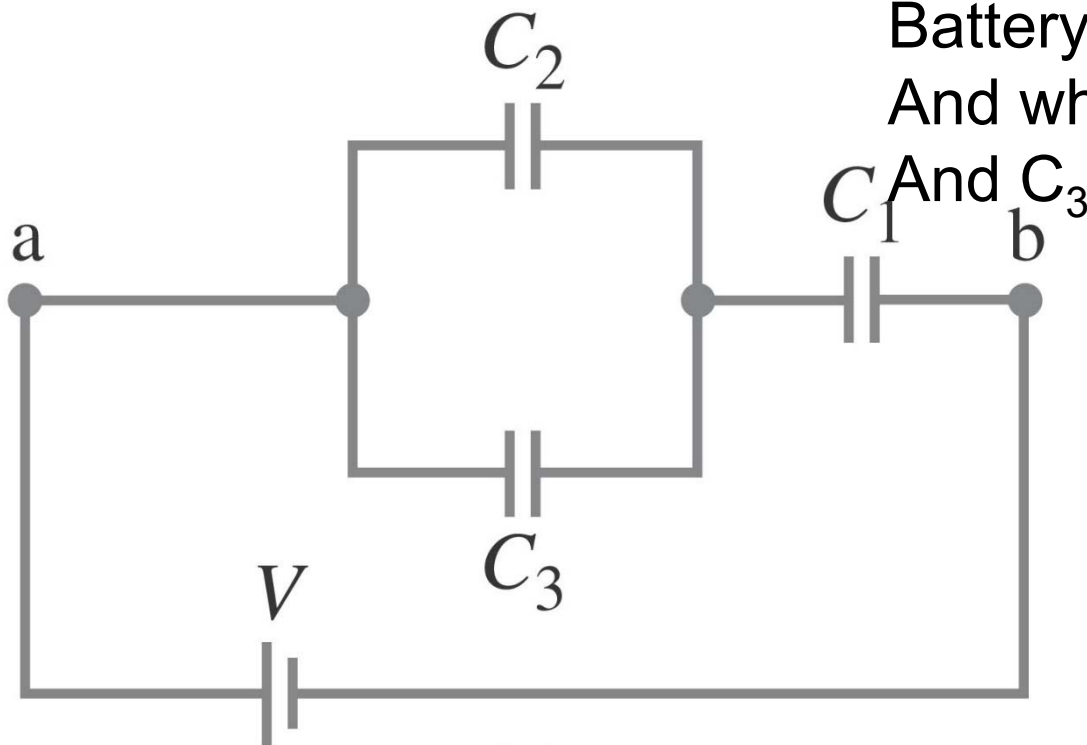
Try now in class! After you get the formula get the C_{eq} assuming all capacitance are the same C !



24-3 Capacitors in Series and Parallel

Example 24-6: Charge and voltage on capacitors. Determine the charge on each capacitor and the voltage across each, assuming $C = 3.0 \mu\text{F}$ for each and the battery voltage is $V = 4.0 \text{ V}$.

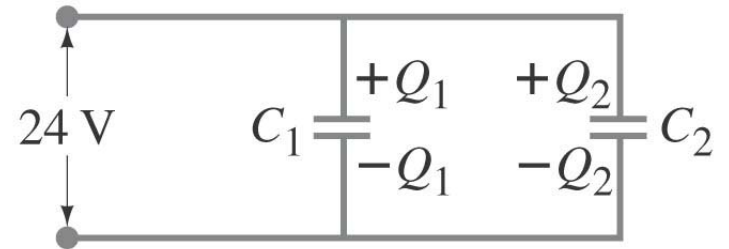
In Class Hint charge from Battery arrives at C_1 . How much And what charge goes to C_2



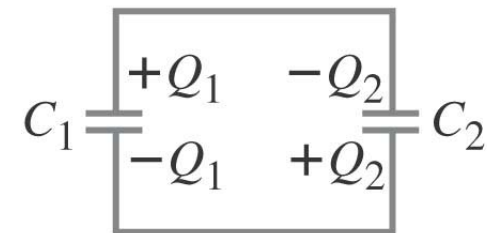
24-3 Capacitors in Series and Parallel

Example 24-7: Capacitors reconnected.

Two capacitors, $C_1 = 2.2 \mu\text{F}$ and $C_2 = 1.2 \mu\text{F}$, are connected in parallel to a 24-V source as shown. After they are charged they are disconnected from the source and from each other and then reconnected directly to each other, with plates of opposite sign connected together. Find the charge on each capacitor and the potential across each after equilibrium is established.



a) Initial configuration.



b) At the instant of reconnection only.



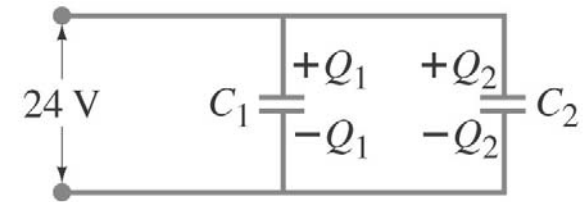
c) A short time later.

We know $C_1=2.2\mu\text{F}$ $C_2 =1.2\mu\text{F}$

$V_{\text{bat}} = 24\text{V}$ find q_1 q_2 and V after equilibrium

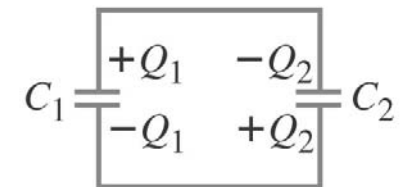
We get $Q_1 = C_1 V_{\text{bat}} = 2.2\mu\text{F} * 24\text{V} = 52.8\mu\text{C}$

$Q_2 = C_2 V_{\text{bat}} = 1.2\mu\text{F} * 24\text{V} = 28.8\mu\text{C}$



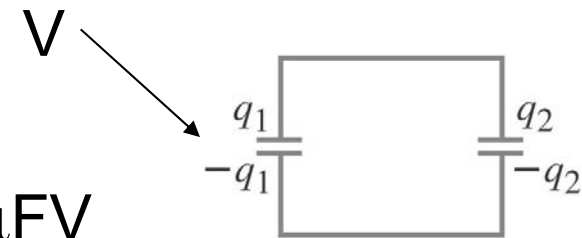
(a) Initial configuration.

Note that the positive charge will migrate to the negative and cancel it out leaving a total positive charge after equilibrium as $Q_1 - Q_2 = 24\mu\text{C} = q_1 + q_2$



(b) At the instant of reconnection only.

**** substituting $q_1 = C_1 V$ and $q_2 = C_2 V$
 $24\mu\text{C} = q_1 + q_2 = C_1 V + C_2 V = (C_1 + C_2) V = 3.4\mu\text{F} V$
 Or $V = 24\mu\text{C} / 3.4\mu\text{F} = 7.06\text{V}$

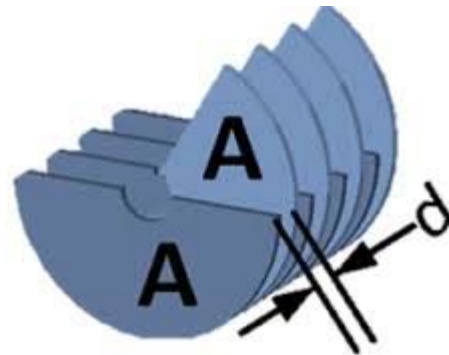


(c) A short time later.

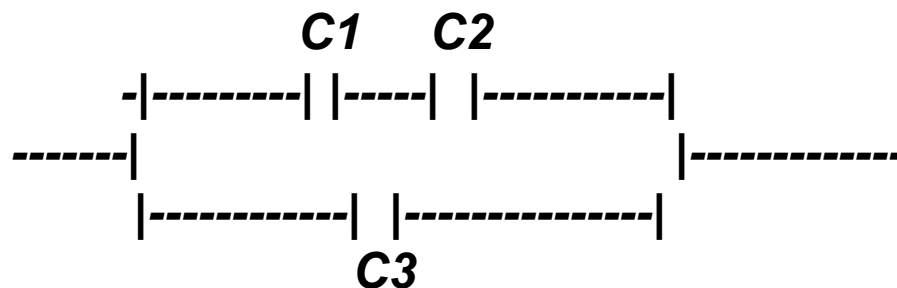
**** gives $q_1 = 15.5\mu\text{C}$ and $q_2 = 8.5\mu\text{C}$

HAND IN HW. Recall by first Sketch, set up equations, solve algebraically then plug in numbers. All answers in Scientific notation

53. Tuning capacitors of the type used in radios have overlapping plates, and the capacitance is changed by varying the amount of overlap, as illustrated here. If each plate has area A and the spacing between the plates is d , what is the capacitance of the unit?



54. Three capacitors $C_1 = 2\mu\text{F}$, $C_2 = 3\mu\text{F}$, and $C_3 = 5\mu\text{F}$ are connected as shown here. What is the equivalent capacitance of this arrangement?



24-4 Electric Energy Storage

A charged capacitor stores electric energy; the energy stored is equal to the work done to charge the capacitor: some dq has to move in but external work has to be done to move it since charge is accumulating on a plate so we must overcome the repulsion by doing work on the dq to get it to the plate.

ROUGH Analogy

$$W = \int F dl = \int dq E \int dl = \int dq E d = \int_0^Q V dq = (1/C) \int_0^Q q dq = (1/2) Q^2 / C$$

Energy stored in a capacity also taking into account $Q=CV$

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 = \frac{1}{2} QV.$$

24-4 Electric Energy Storage

Conceptual Example 24-9: Capacitor plate separation increased.

A parallel-plate capacitor carries charge Q and is then disconnected from a battery. The two plates are initially separated by a distance d . Suppose the plates are pulled apart until the separation is $2d$. How has the energy stored in this capacitor changed?

Think about it... $C=?$ $U=?$

*$C = \epsilon_0 A / d$ and $U = 1/2 Q^2 / C$ so $d \rightarrow 2d \rightarrow C$ goes down by 2
then since Q does not change U goes up by 2*

We did work to separate them so potential E goes UP!

****24-4 Electric Energy Storage

Example 24-8: Energy stored in a capacitor.

A camera flash unit stores energy in a $150\text{-}\mu\text{F}$ capacitor at 200 V . (a) How much electric energy can be stored? (b) What is the power output if nearly all this energy is released in 1.0 ms ?

IN CLASS!!

Ans
 3J
& 3000W



24-4 Electric Energy Storage

Example 24-10: Moving parallel capacitor plates.

The plates of a parallel-plate capacitor have area A , separation x , and are connected to a battery with voltage V . While connected to the battery, the plates are pulled apart until they are separated by $3x$. (a) What are the initial and final energies stored in the capacitor? (b) How much work is required to pull the plates apart (assume constant speed)? (c) How much energy is exchanged with the battery?

*(a) Since we have A and x we know $C = \epsilon_0 A/x$ and we have V so we can use $U_i = 1/2 CV^2 = 1/2 \epsilon_0 AV^2/x$ $U_f = 1/2 \epsilon_0 AV^2/3x$ Potential energy decreases $\Delta U_{cap} = U_f - U_i = -\epsilon_0 AV^2/3x$ note: negative
Since C decreases by 3 so capacity of Q drops so some Q will leave the plates with same V attached.*

*Since we are doing work on the plates, ie adding energy but U dropped
Where did the positive energy go in this system?*

*Answer is the Battery: See the text for the calculation of $W = \epsilon_0 AV^2/3x$
And COE means $W = \Delta U_{cap} + \Delta U_{bat}$ or $\Delta U_{batt} = W - \Delta U_{cap}$ or
 $\Delta U_{batt} = \epsilon_0 AV^2/3x + \epsilon_0 AV^2/3x = 2\epsilon_0 AV^2/3x$*

24-4 Electric Energy Storage

The energy density, defined as the energy per unit volume, is the same no matter the origin of the electric field: It is useful to consider the energy stored in a capacitor as stored in the electric field.

For || plate $C = \epsilon_0 A/d$ and $V = Ed$ then $U = 1/2 CV^2$

Becomes $U = 1/2 \epsilon_0 A/d \times (Ed)^2 = 1/2 \epsilon_0 E^2 Ad$ ($Ad = \text{Vol}$)

So the energy /volume and energy density considered in the Field is $U/\text{Vol} = u = 1/2 \epsilon_0 E^2$ Can be shown true for all E fields!!

In sum

$$u = \text{energy density} = \frac{1}{2} \epsilon_0 E^2.$$

IN CLASS CHECK UNITS WORK!!

The sudden discharge of electric energy can be harmful or **fatal**. Capacitors can retain their charge **indefinitely** even when disconnected from a voltage source – be careful!

24-4 Electric Energy Storage

Heart defibrillators use electric discharge to “jump-start” the heart, and can save lives.

Few thousand volts

To stop Ventricular fibrillation



HAND IN HW. Recall by first Sketch, set up equations, solve algebraically then plug in numbers. All answers in Scientific notation

55. A 1.0- μ F capacitor is charged to 100 V.

- (a) How much charge does it store?**
- (b) How much energy does it store?**
- (c) A 2- μ F capacitor is charged to 50 V.**
- (d) How much charge and energy does it**

56. In order to create a nuclear fusion reaction, of the type that generates energy in the Sun, it is necessary to raise the nucleons involved to very high temperatures. This is because the positively charged particles experience a large electrostatic repulsion. Many techniques have been tried to accomplish reaching the required high temperatures. One such approach is the NOVA experiment being carried out at the Lawrence Livermore Laboratory in California. Here, small pellets containing the nuclear reactants are irradiated simultaneously from all sides by many high powered lasers. The plan is to raise the pellets to such high temperatures that nuclear fusion occurs.

A typical laser pulse generates about 10^{14} W of power for a duration of 10^{-9} s. This is a power level about 100 times as great as that of all the world's power plants.

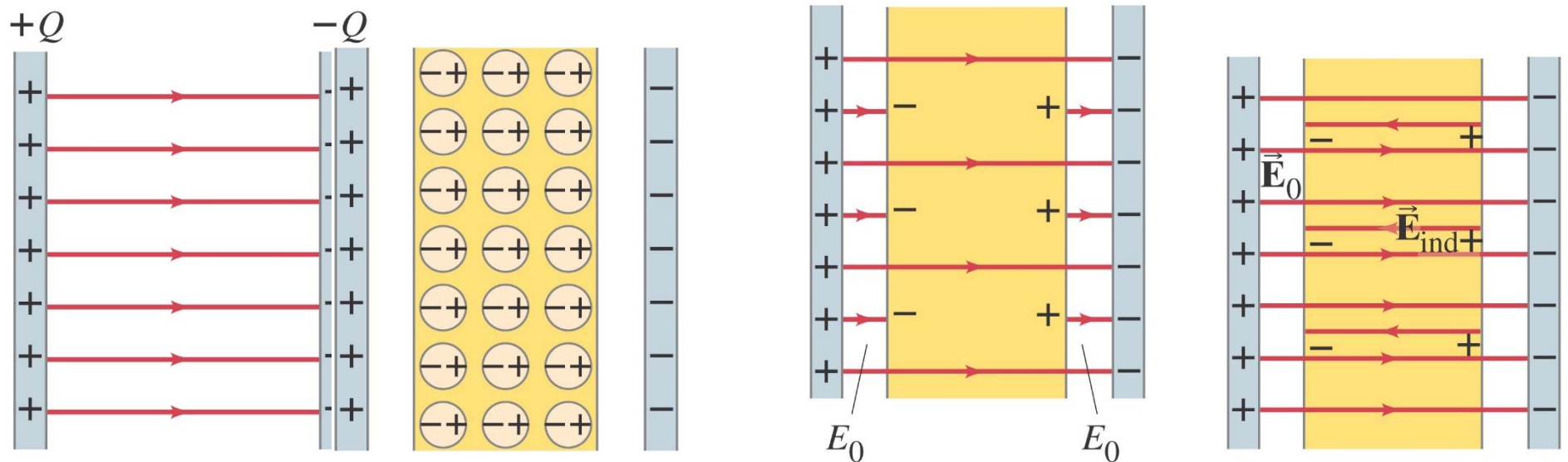
- (a) How much energy is generated in one pulse of the lasers?**
- (b) To what voltage must the capacitor bank supplying the energy be charged if the total capacitance is 0.25 F and 0.15% of the electrical energy appears as light in the lasers?**

Dielectrics: Molecular Level

Placing an insulating sheet between the plates will increase the capacitance of a capacitor, as well as, making it difficult for charge to flow across the gap by a breakdown in the gap (a dielectric breakdown).

The molecules in a dielectric, when in an external electric field, tend to become oriented in a way that reduces the external field. E is reduced by a factor K called the dielectric constant for the particular material. Q is constant.

*New $E_D = E_0/K$ so $V_D = E_D d = E_0 d /K \rightarrow V_0 /K$ but $Q = CV_D$ so to keep Q constant $Q = C_0 V_0 /K \rightarrow C_0 = KC_0$ and thus $Q = C_0 V_0 = Q_0$ ie Q constant
But a new $C = KC_0$ means in general $Q = CV_0 = KC_0 V_0$ so $Q = KQ_0$*



24-5 Dielectrics

A dielectric is an insulator, and is characterized by a dielectric constant K .

Capacitance of a parallel-plate capacitor filled with dielectric:

$$C = K\epsilon_0 \frac{A}{d} \quad [\text{parallel-plate capacitor}]$$

Using the dielectric constant, we define the permittivity of a material!:

$$\epsilon = K\epsilon_0.$$

so $C = \epsilon A/d$

Energy density stored in the Electric field goes up also as would be expected and $u = 1/2 K\epsilon_0 E^2 = 1/2 \epsilon E^2$

24-5 Dielectrics

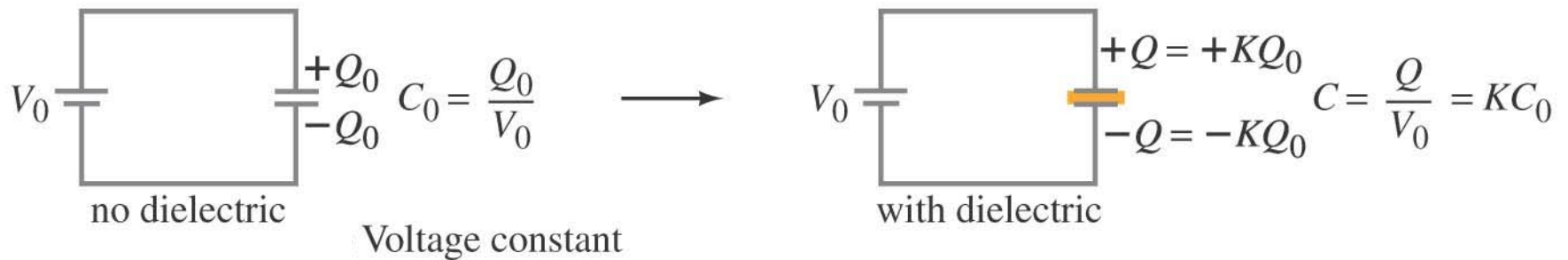
TABLE 24-1
Dielectric Constants (at 20°C)

Material	Dielectric constant K	Dielectric strength (V/m)
Vacuum	1.0000	
Air (1 atm)	1.0006	3×10^6
Paraffin	2.2	10×10^6
Polystyrene	2.6	24×10^6
Vinyl (plastic)	2-4	50×10^6
Paper	3.7	15×10^6
Quartz	4.3	8×10^6
Oil	4	12×10^6
Glass, Pyrex	5	14×10^6
Porcelain	6-8	5×10^6
Mica	7	150×10^6
Water (liquid)	80	
Strontium titanate	300	8×10^6

Dielectric strength is the maximum field strength a dielectric can experience without breaking down.

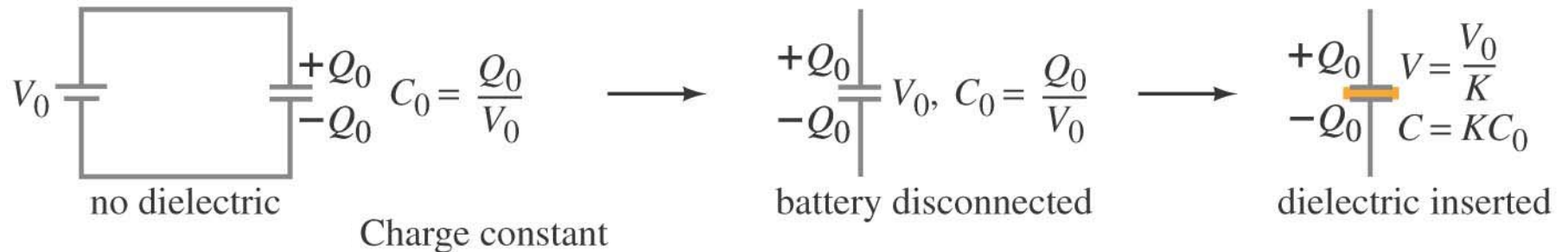
24-5 Dielectrics

Text examples: two experiments where we insert and remove a dielectric from a capacitor. In the first, the capacitor is connected to a battery, so the voltage remains constant. The capacitance increases, and therefore the charge on the plates increases as well.



24-5 Dielectrics

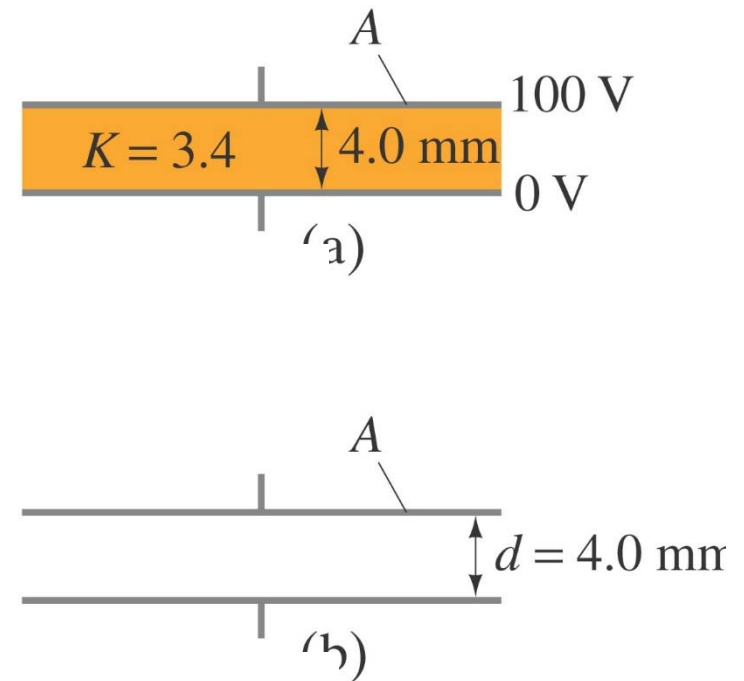
In this second experiment, we charge a capacitor, disconnect it, and then insert the dielectric. In this case, the charge remains constant. Since the dielectric increases the capacitance, the potential across the capacitor drops.



24-5 Dielectrics

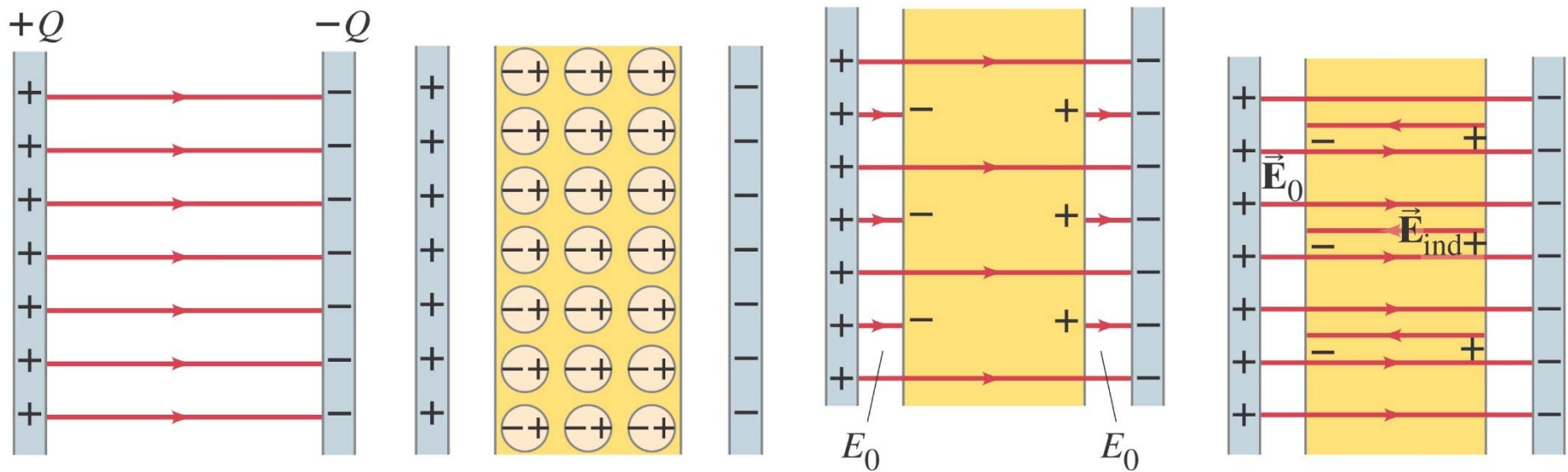
Example 24-11: Dielectric removal.

A parallel-plate capacitor, filled with a dielectric with $K = 3.4$, is connected to a 100-V battery. After the capacitor is fully charged, the battery is disconnected. The plates have area $A = 4.0 \text{ m}^2$ and are separated by $d = 4.0 \text{ mm}$. (a) Find the capacitance, the charge on the capacitor, the electric field strength, and the energy stored in the capacitor. (b) The dielectric is carefully removed, without changing the plate separation nor does any charge leave the capacitor. Find the new values of capacitance, electric field strength, voltage between the plates, and the energy stored in the capacitor.



24-6 Molecular Description of Dielectrics

The molecules in a dielectric, when in an external electric field, tend to become oriented in a way that reduces the external field.



24-6 Molecular Description of Dielectrics

This means that the electric field within the dielectric is less than it would be in air, allowing more charge to be stored for the same potential. This reorientation of the molecules results in an induced charge – there is no net charge on the dielectric, but the charge is asymmetrically distributed.

The magnitude of the induced charge depends on the dielectric constant:

$$Q_{\text{ind}} = Q \left(1 - \frac{1}{K} \right).$$

HAND IN HW. Recall by first Sketch, set up equations, solve algebraically then plug in numbers. All answers in Scientific notation

57. A parallel plate capacitor is half filled with a dielectric with $k = 2.5$. What fraction of the energy stored in the charged capacitor is stored in the dielectric? HINT: it like a parallel connection so how would you get the capacitance and what is the relationship to Energy stored. See figure below.

